Classical Mechanics

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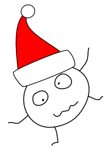
Exercise sheet 10 **Due: Thursday January 13 at 24:00**

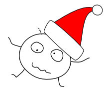


1 Generalized Noether

The Noether-theorem that we employed in the previous exercise can be generalized. Recall from the lecture that a family of trajectories $\vec{q}^{(s)}(t)$ with $\vec{q}^{(0)}(t) = \vec{q}(t)$ satisfies the conditions for the generalized Noether theorem if there exists a function $\vec{\Delta}$ and a function f such that

$$\frac{d}{ds}|_{s=0}\vec{q}^{(s)}(t) = \vec{\Delta}(\vec{q}(t), \dot{\vec{q}}(t), t) \tag{1}$$

$$\frac{d}{ds}\Big|_{s=0} L(\vec{q}^{(s)}(t), \dot{\vec{q}}^{(s)}(t), t) = \frac{d}{dt} f(\vec{q}(t), \dot{\vec{q}}(t), t).$$



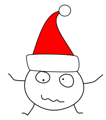
Suppose that we have a particle of mass m that lives on a line ($\mathbb{R}$) with coordinate q. This particle is affected by a time-dependent potential $V(q,t) = g(t)q$, for some (nice) function g.

a) *Determine the Lagrangian for the particle with respect to the coordinate q.* **(2 points)**

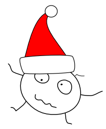
b) Consider the family of trajectories $q^{(s)}(t) = q(t) + s$. *Find functions Δ and f that satisfy the conditions of the generalized Noether-theorem in (1).* **(3 points)**

c) *Determine the conserved quantity corresponding to the transformation in b).* **(3 points)**

d) As an alternative to the generalized Noether-theorem, one can show that the quantity in c) is conserved by directly using the equation of motion of the particle. *Do that derivation.* **(2 points)**



2 From Lagrange to Hamilton



For the following Lagrangians, derive the Hamilton functions, and the Hamilton equations.

a)

$$L(\theta, \dot{\theta}) = \frac{m}{2} R^2 \dot{\theta}^2 + \frac{m}{2} R^2 \Omega^2 \sin^2\theta + mgR\cos\theta$$

(3 points)

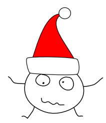
b)

$$L(r, \theta, \dot{r}, \dot{\theta}) = \frac{m}{2}\left(1 + \frac{\alpha^2}{r^6}\right)\dot{r}^2 + \frac{m}{2} r^2 \dot{\theta}^2 + \frac{mg\alpha}{2r^2}$$

(4 points)

c)

$$L(\vec{r}, \dot{\vec{r}}) = \frac{m}{2}\dot{\vec{r}}^2 - e\phi(\vec{r}, t) + e\vec{A}(\vec{r}, t) \cdot \dot{\vec{r}},$$



(3 points)

Comment: This Lagrangian comes from an earlier exercise. Can you see which one?