Classical Mechanics

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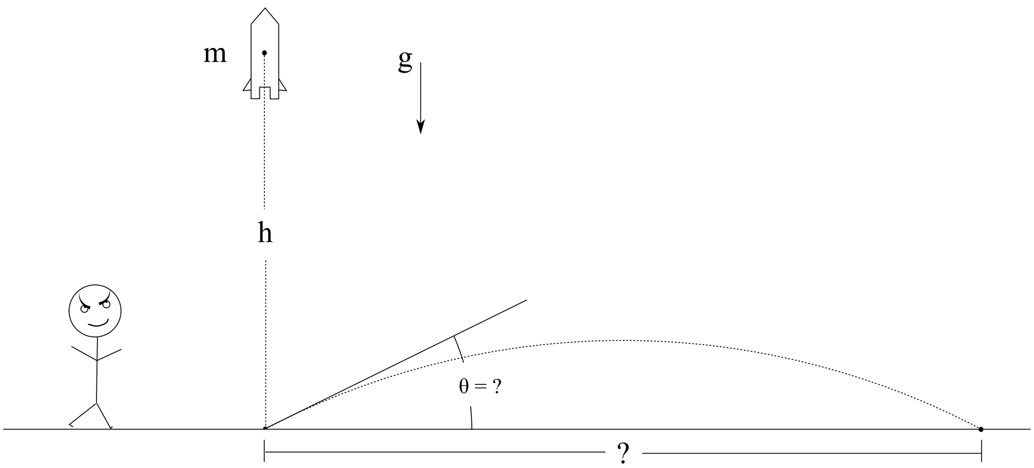
Exercise sheet 1 Due: Thursday October 21 at 24:00

1 Dictator as career choice: Rocket science

Presumably you have been thinking of what to do after your physics studies. One career choice would be to become dictator of some random country. With solid knowledge in classical mechanics you can threaten the entire world with your powerful rockets.



Suppose that you want to test the nice new rocket that your minions have built. It is a smart move to direct the rocket vertically[1]. In that way you look threatening, but do not directly provoke your neighbours. To simplify things we assume that all energy is released at the initial moment when the rocket is fired, so that after the initial burst there is no additional propulsion[2]. Moreover, we assume that there is no friction, that gravity is independent of vertical position, and that we can ignore the curvature (as well as rotation) of Earth[3].

Suppose that the rocket has mass m and reaches the distance h above ground when fired vertically. How far could it reach horizontally if it instead is aimed at an angle? What is the optimal angle?

Hint: For the vertical launch, what is the initial speed required to reach h? Use Newton's second law to find the trajectory when the rocket instead is fired at an angle. How far can it reach as a function of θ?

(7 points)

Comment: The purpose of this exercise is to deal with concepts such as conservation of energy, Newton's second law, and equations of motion.

[1] *Dictatorship for dummies*, 2nd Edition, Wiley (2011).

[2] To be honest, we are here modeling a cannon rather than a rocket, but cannons are much less fashionable among dictators these days.

[3] These are of course not very good assumptions if you want your rocket to reach some big country on the other side of Earth.

2 Alice and Bob collide projectiles

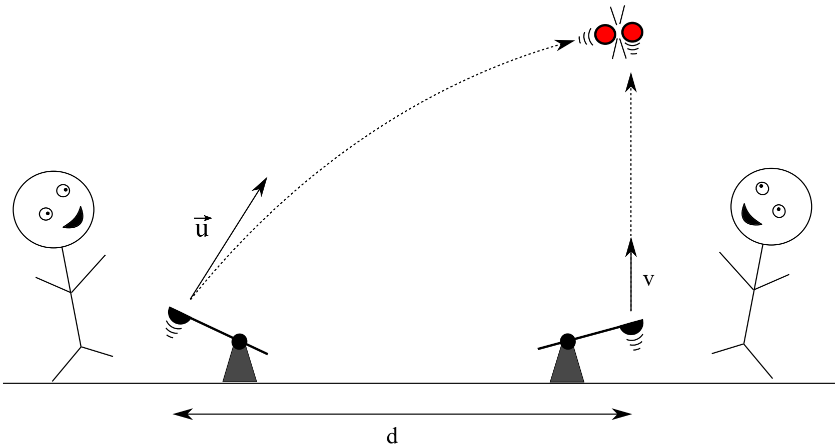


Figure 1: Students Alice and Bob derive an absurd amount of joy from colliding things in the air.

Students Alice and Bob have as a hobby to shoot things into the air with catapults. In order to be as spectacular as possible they would like to make the projectiles collide at the very top of their trajectories. Alice shoots her projectile right up vertically, initially at speed v. Bob's catapult stands at a distance d. We assume that Alice and Bob's catapults fire at the same time, that the projectiles start at the same distance from the ground, and that there is no friction.

a) *By what initial velocity vector $\vec{u}$ should Bob shoot his projectile, such that it collides with Alice's when both projectiles are at the maximum height of their trajectories?* **(4 points)**

b) *For a fixed d, what should v be such that $|\vec{u}|$ is minimal?* **(2 points)**

Comment: The purpose of this exercise is to find and solve the relevant equations of motion of a system, and how this can be used to tune parameters in order to reach a certain objective.

3 Block and wedge

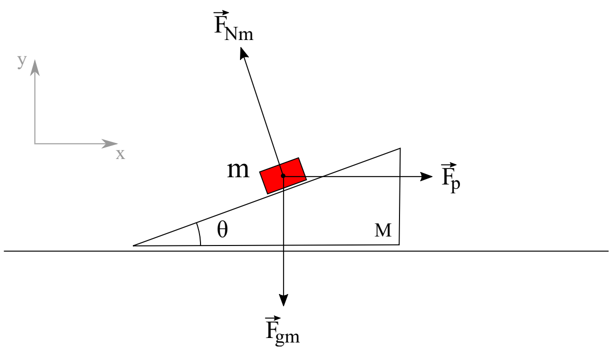


Figure 2: A block (in red) of mass m slides without friction on a wedge with mass M, which in turn slides without friction on a horizontal surface. Here we have have included the forces acting on the block, which are gravity $\vec{F}_{gm}$, the normal force $\vec{F}_{Nm}$ from the wedge onto the block, and an external horizontal force $\vec{F}_p$ by which we pull the block.

A block of mass m can slide without friction on a wedge of mass M, which in turn slides without friction on a horizontal surface. The angle of the wedge is $0 < \theta < \pi/2$. On the block we apply an external force $\vec{F}_p = F_p\hat{x}$ in the horizontal direction, with $F_p \geq 0$. The block is moreover affected by gravity $\vec{F}_{gm} = -mg\hat{y}$, and a normal force $\vec{F}_{Nm}$ from the wedge (that prevents the block from sliding through the surface of the wedge). Here $\hat{x}$ and $\hat{y}$ are the unit-vectors in the horizontal x-direction and the vertical y-direction.

a) *Which forces act on the wedge?* (It is enough with a qualitative answer, where you tell which are the forces and what directions they have, but not the magnitudes.)

Remark: We only apply the additional force $\vec{F}_p$ to the block.

(2 points)

b) Assume that the block and wedge initially do not move relative to each other. *How large does F_p have to be in order for the block and wedge to accelerate in the x-direction, without moving relative to each other?*

Hint: It is often a good idea to consider what forces act on each object separately. You will end up with a system of equations.

(5 points)

Comment: The primary purpose of this exercise is to reason about forces in composite systems.