Lecturer: David Gross. Exercises: Mariami Gachechiladze, Lukas Franken.
E-mail for submissions/questions: marigachi@gmail.com

(13) Joint measurability and covariance **20 points**

In the lecture we considered a parametrisation of POVM effects:

$$E(a_0, \vec{a}) = \frac{1}{2}(a_0 \mathbb{1} + a_x \sigma_x + a_y \sigma_y + a_z \sigma_z),$$

where all a_i's are real coefficients. The special case when $a_y = 0$, and $a_0 = 1$, $E(1, \vec{a})$ is called covariant.

Take two outcome POVMs such that their effects, $E(a_0, \vec{a})$ and $E(b_0, \vec{b})$ are covariant. Show that if the following geometric inequality holds,

$$||\vec{a} + \vec{b}||_2 + ||\vec{a} - \vec{b}||_2 \leq 2$$

then these POVMs are jointly measurable.