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(5) Experimentally testable state-independent quantum contextuality 10 points

In the lecture we looked at Peres-Mermin square and established the contradiction for the deterministic ± 1 assignments and quantum mechanical predictions.

$$\begin{array}{ccc} A = \sigma_Z \otimes \mathbb{1} & B = \mathbb{1} \otimes \sigma_Z & C = \sigma_Z \otimes \sigma_Z \\ a = \mathbb{1} \otimes \sigma_X & b = \sigma_X \otimes \mathbb{1} & c = \sigma_X \otimes \sigma_X \\ \alpha = \sigma_Z \otimes \sigma_X & \beta = \sigma_X \otimes \sigma_Z & \gamma = \sigma_Y \otimes \sigma_Y \end{array}$$

Here we will try to convert this contradiction to the experimentally testable inequality. We can again consider the products of the observables on each row, i.e., ABC,abc, $\alpha\beta\gamma$ and the same for the columns, respectively. Recall that each observable has a deterministic ± 1-valued assignment.

a) What is the maximum value (and why?) that the following inequality can attain for any non-contextual hidden variable (NCHV) theory in which the observables have deterministic results

$$\langle \chi \rangle = \langle ABC \rangle + \langle abc \rangle + \langle \alpha\beta\gamma \rangle + \langle Aa\alpha \rangle + \langle Bb\beta \rangle - \langle Cc\gamma \rangle ? \tag{1}$$

(b) We now consider any quantum state $|\psi\rangle \in H^{(4)}$, which lives in $d = 4$ dimensional Hilbert space. In Eq (1) for the values of quantum mechanical observables use the assignments given in Peres-Mermin square above and then calculate the expectation value of the operator χ for any quantum state $|\psi\rangle$.

$$\langle \psi \chi | \psi \rangle = ? \tag{2}$$

Does the result violate the value obtained for NCHV theories?

(⋆) Help an evil detector!

A fraudulent detector needs your help to come up with a strategy to falsify the value of CHSH inequality in classical physics. In reality, CHSH = 2. But the detector can sometimes swallow photons, in order to violate that bound. Note that such runs of experiments can be discarded. So, what shall the strategy be?

You do not get points for this exercise, but try not to miss out the fun of solving the riddle.