Sheet #4: FOUNDATIONS OF QUANTUM MECHANICS Due: 18.05.20, 13:00
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(6) Communication complexity and PR-boxes **15 points**

Consider a well-known CHSH scenario: two parties A and B share a distributed physical resource. Each party can independently perform one out of two measurements $x \in \{0,1\}$ and $y \in \{0,1\}$ on their part of the system and obtain the binary outcomes respectively, a_x and b_y. Here $\oplus$ is mod 2 summation.

(a) Show that the following inequalities/bounds hold for different theories: **3 points**

$$\text{Any LHV theory} \sum_{x,y\in\{0,1\}} P(a_x \oplus b_y = x \cdot y) \leq 3 \tag{1}$$

$$\text{Quantum theory} \sum_{x,y\in\{0,1\}} P(a_x \oplus b_y = x \cdot y) \leq 2 + \sqrt{2}. \tag{2}$$

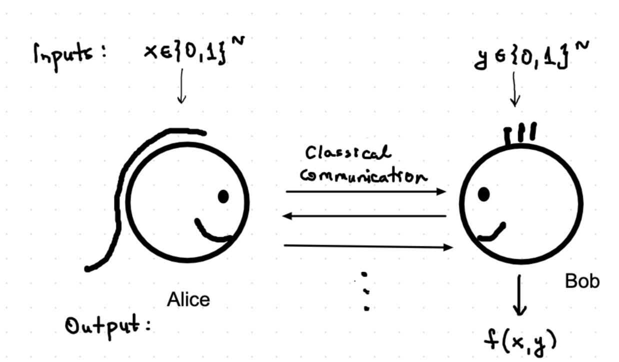
$$\text{Post-quantum theory} \sum_{x,y\in\{0,1\}} P(a_x \oplus b_y = x \cdot y) = 4. \tag{3}$$

The final value is calculated using PR boxes, which exhibits the following correlations.

$$P(a_x = 0, b_y = 0) = P(a_x = 1, b_y = 1) = \frac{1}{2}, \quad xy \in \{00, 01, 10\}. \tag{4}$$

$$P(a_x = 0, b_y = 1) = P(a_x = 1, b_y = 0) = \frac{1}{2}, \quad xy = 11. \tag{5}$$

(b) We look at the consequences of superstrong nonlocality for the theory of communication complexity, which describes how much communication is needed to evaluate a distributed function f. See the drawing below and the description of the scenario. **10 points**



Scenario: consider a Boolean function $f(x,y) = \sum_i^N x_i \cdot y_i$ which has as input two N-bit strings $x, y \in \{0,1\}^N$. Now if Alice and Bob share PR boxes (Eqs. (3) and (4)), how many bits do they have to exchange in order to determine the function value (the inner product of their respective strings) if A possesses the x string and B the y-string?

(c) It is known that the optimal communication for both, the classical and quantum cases is N bits (we can discuss this in the class). Compare your results with these statements and argue about them. **2 points**