Sheet #9: FOUNDATIONS OF QUANTUM MECHANICS Due: 29.06.20, 13:00

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(12) Measurements and decoherence — 20 points

Weak, or incomplete measurements are those that do not project an initially mixed state only onto pure states that is, they are measurements whose measurement operators are not all rank 1, and thus do not provide the observer with full information about the state of the system following the measurement. Consider a system that is initially in the state

$$\rho = p|0\rangle\langle 0| + (1-p)|1\rangle\langle 1|. \tag{1}$$

Now consider the two-outcome measurement with projection operators given by

$$A_+ = \sqrt{k}|\alpha_+\rangle\langle\alpha_+| + \sqrt{1-k}|\alpha_-\rangle\langle\alpha_-|, \tag{2}$$

$$A_- = \sqrt{1-k}|\alpha_+\rangle\langle\alpha_+| + \sqrt{k}|\alpha_-\rangle\langle\alpha_-|, \tag{3}$$

where $|\alpha_+\rangle = \sqrt{\alpha}|0\rangle + \sqrt{1-\alpha}|1\rangle$ and $|\alpha_-\rangle = \sqrt{1-\alpha}|0\rangle - \sqrt{\alpha}|1\rangle$ and $0 \leq \alpha, k \leq 1$.

(a) Write a computer program in your favourite language (e.g. Mathematica) to calculate the increase in the purity of the state $\mathrm{Tr}\,(\tilde{\rho}^2)$ due to the measurement, averaged over the two measurement outcomes. That is $\tilde{\rho} = \sum_\pm p_\pm \tilde{\rho}_\pm = \sum_\pm A_\pm \rho A_\pm^\dagger$. Fix $p = 0, p = 0.2, p = 0.4, p = 0.5$ and plot purity for $0 \leq \alpha, k \leq 1$.

(b) For fixed values of p and k, what value of α gives the maximum average increase in the purity?

(c) Draw the initial state ρ on the Bloch sphere, along with the states $|\alpha_\pm\rangle$ for the measurement that gives the maximum increase in purity.