Observational Causal Inference

Lost in Stats: Pitfalls of the Scientific Method

University of Cologne

Mariami Gachechiladze, 24.11.2021

Outline

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Part 1:

Simpson's paradox and its resolution

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Part 2:

Observational Causal Inference

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Simpson's paradox and its resolution

Recap: Simpson's paradox

	Combined	E	$\neg E$		Recovery Rate
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		36	44	80	

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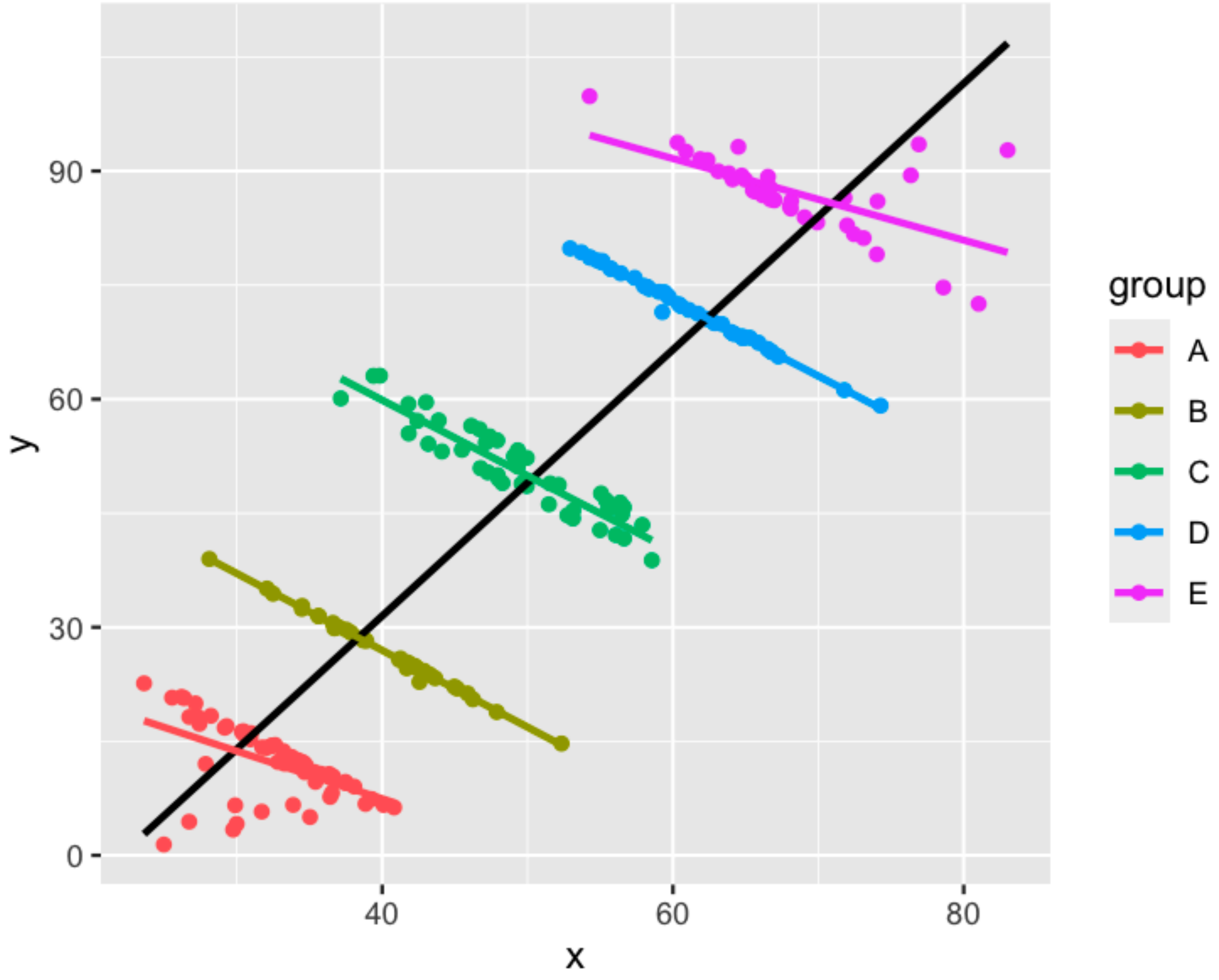

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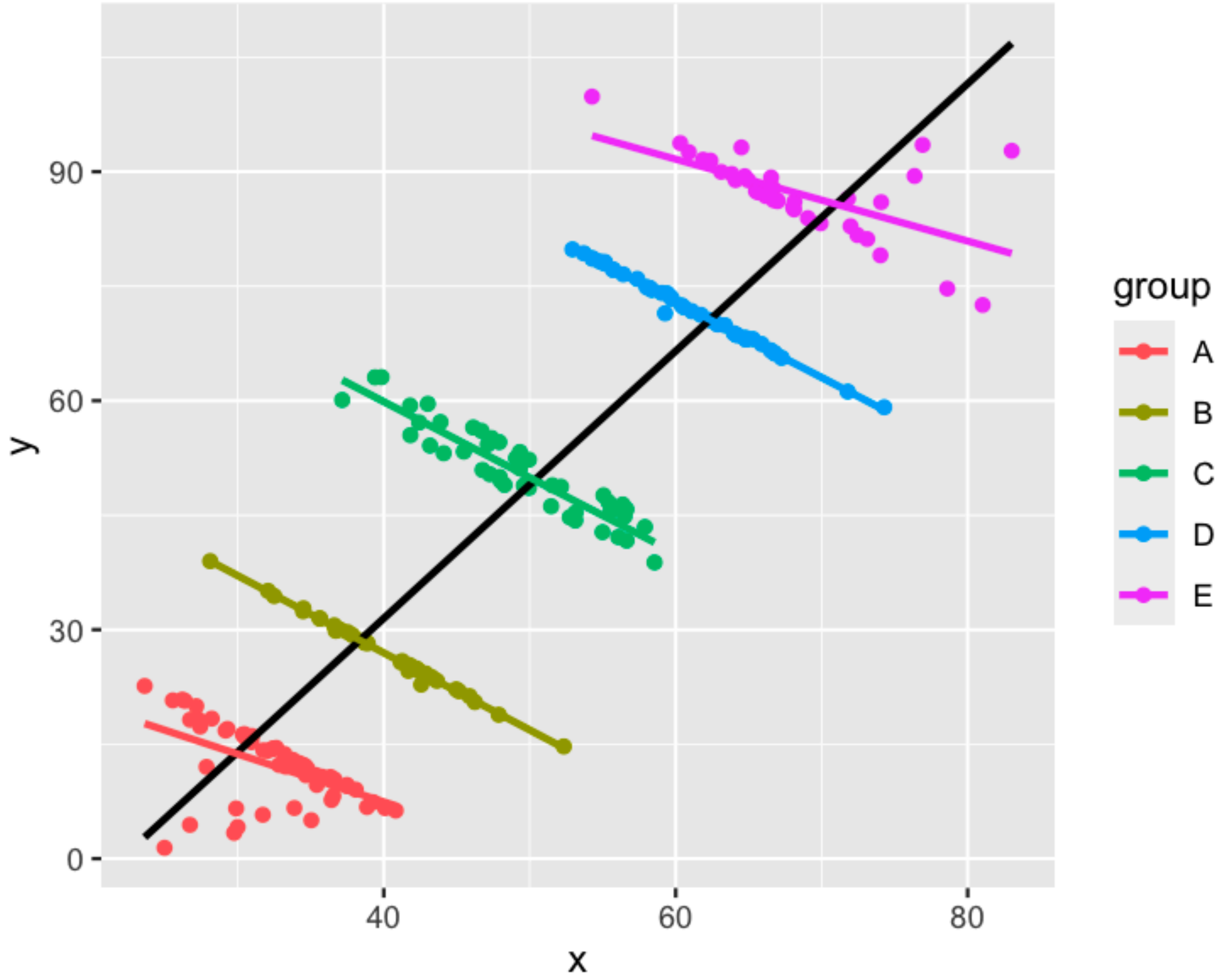
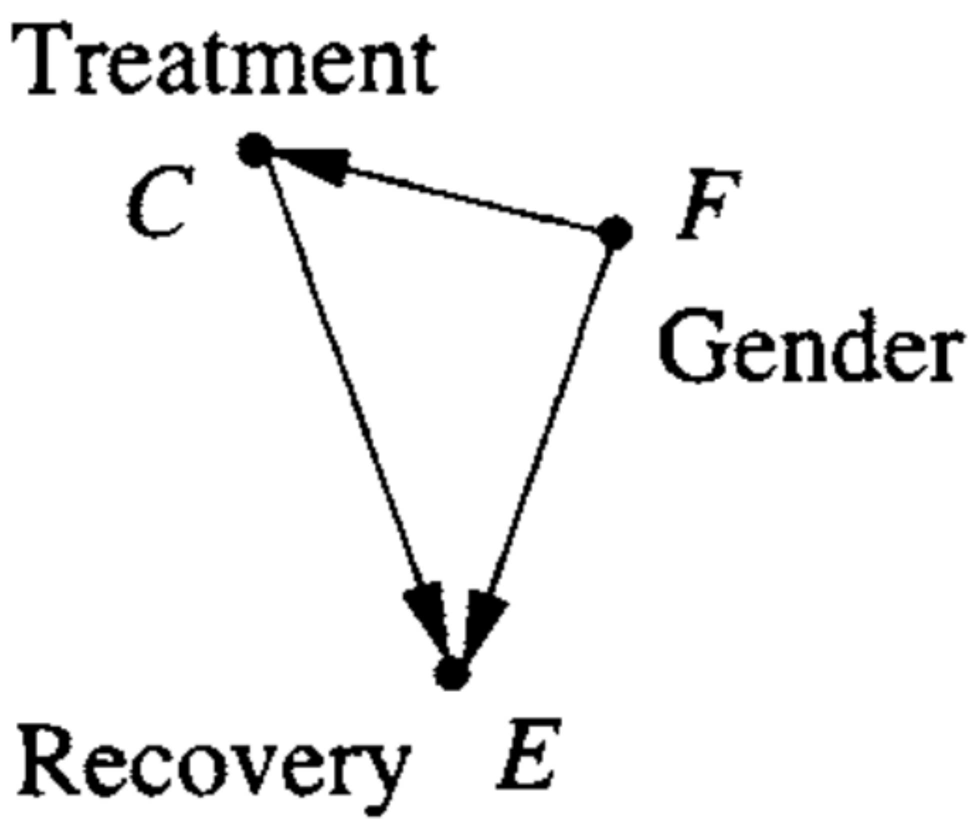


Question: Shall we give a drug to a next patient or not?

Statistics alone cannot suffice



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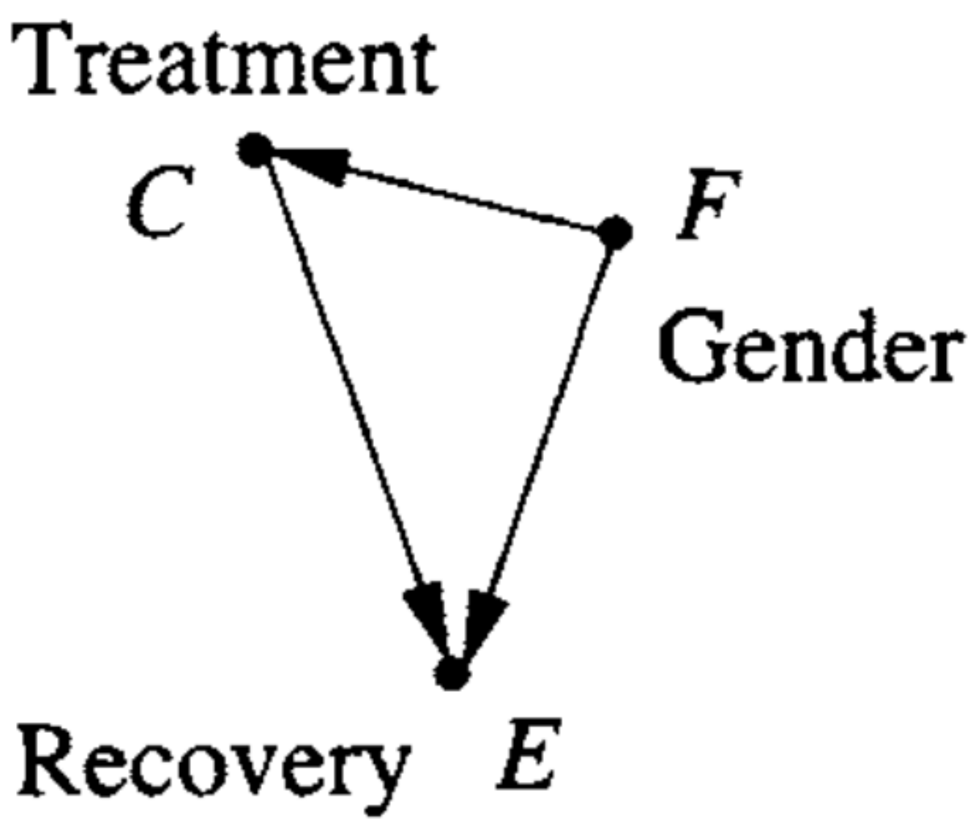

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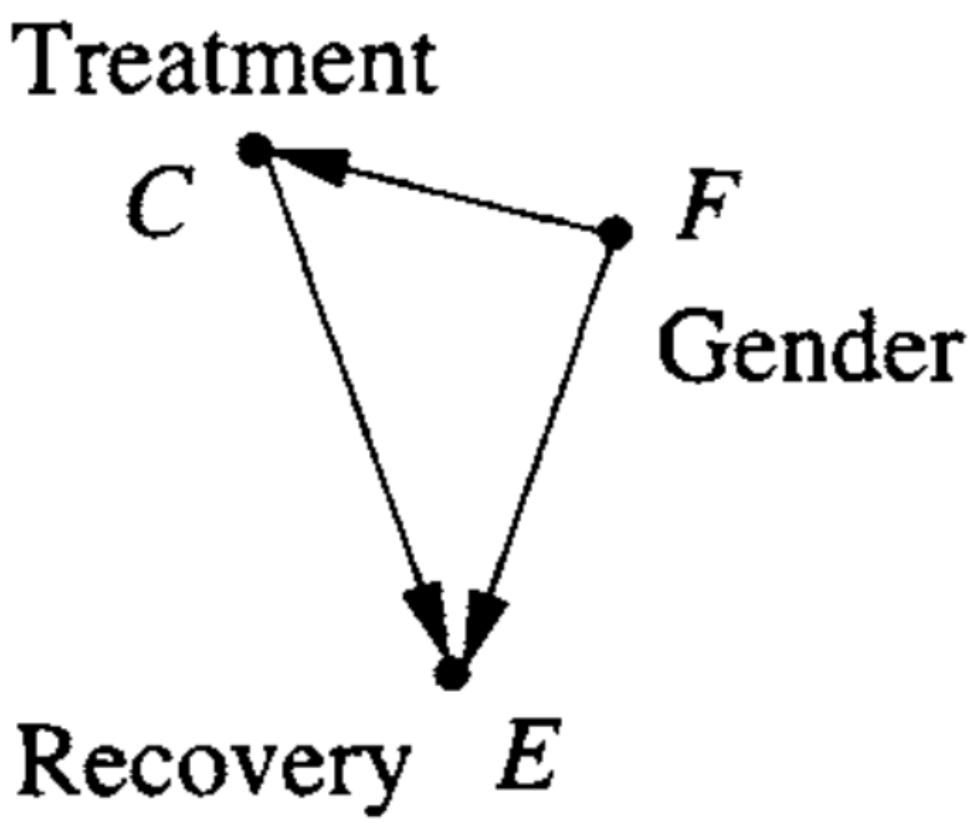

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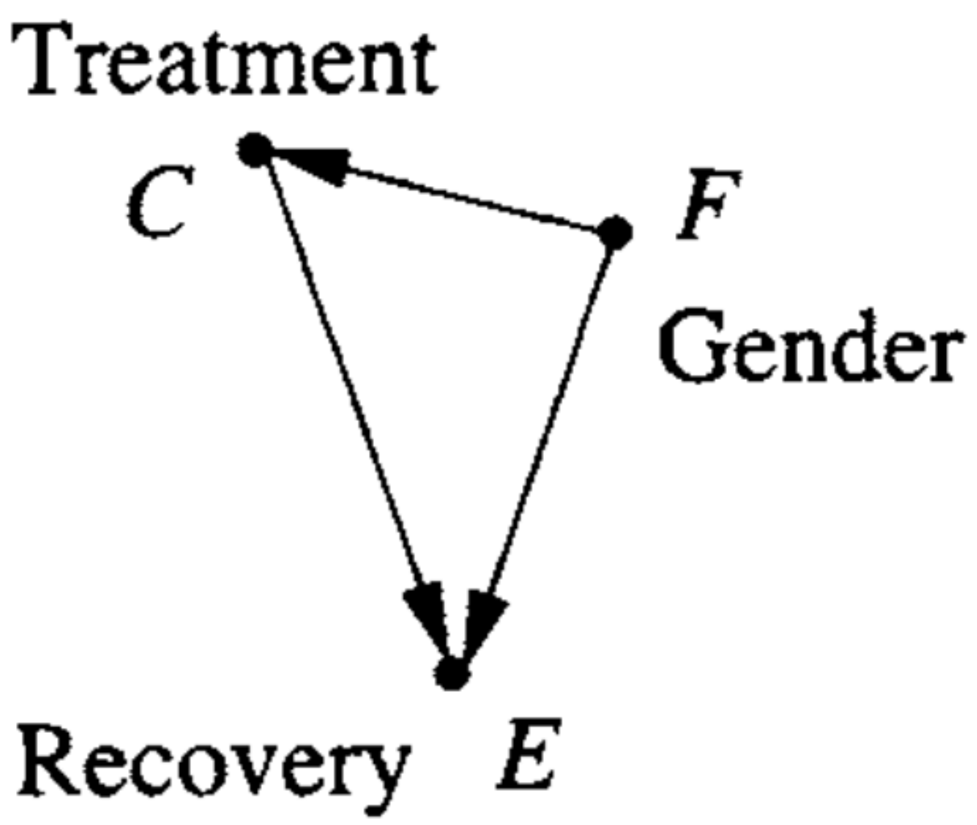

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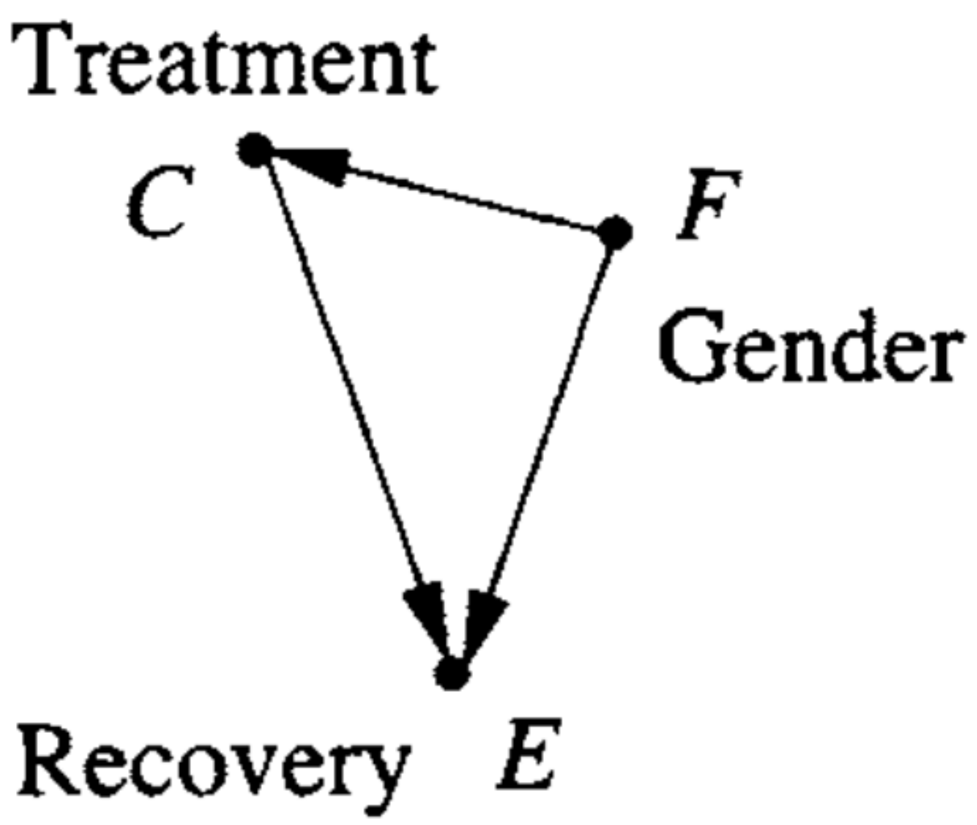

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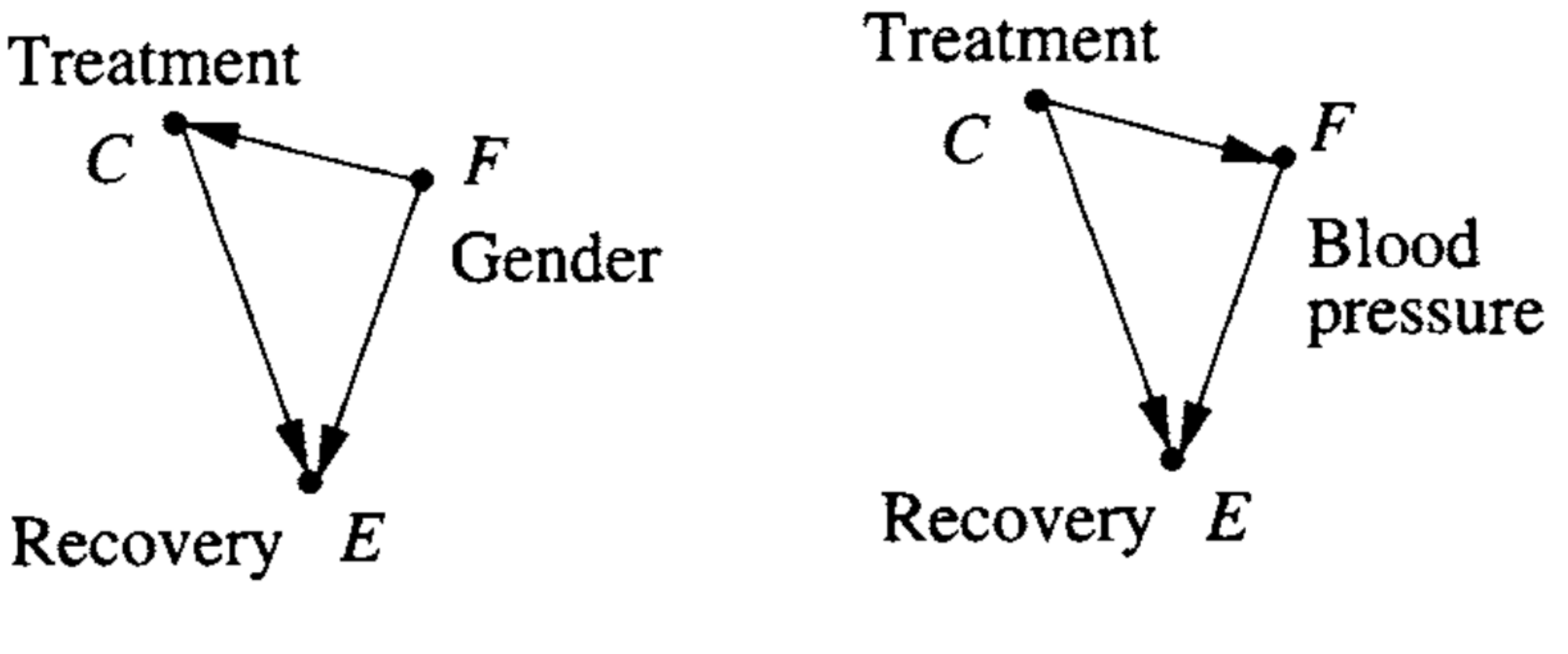

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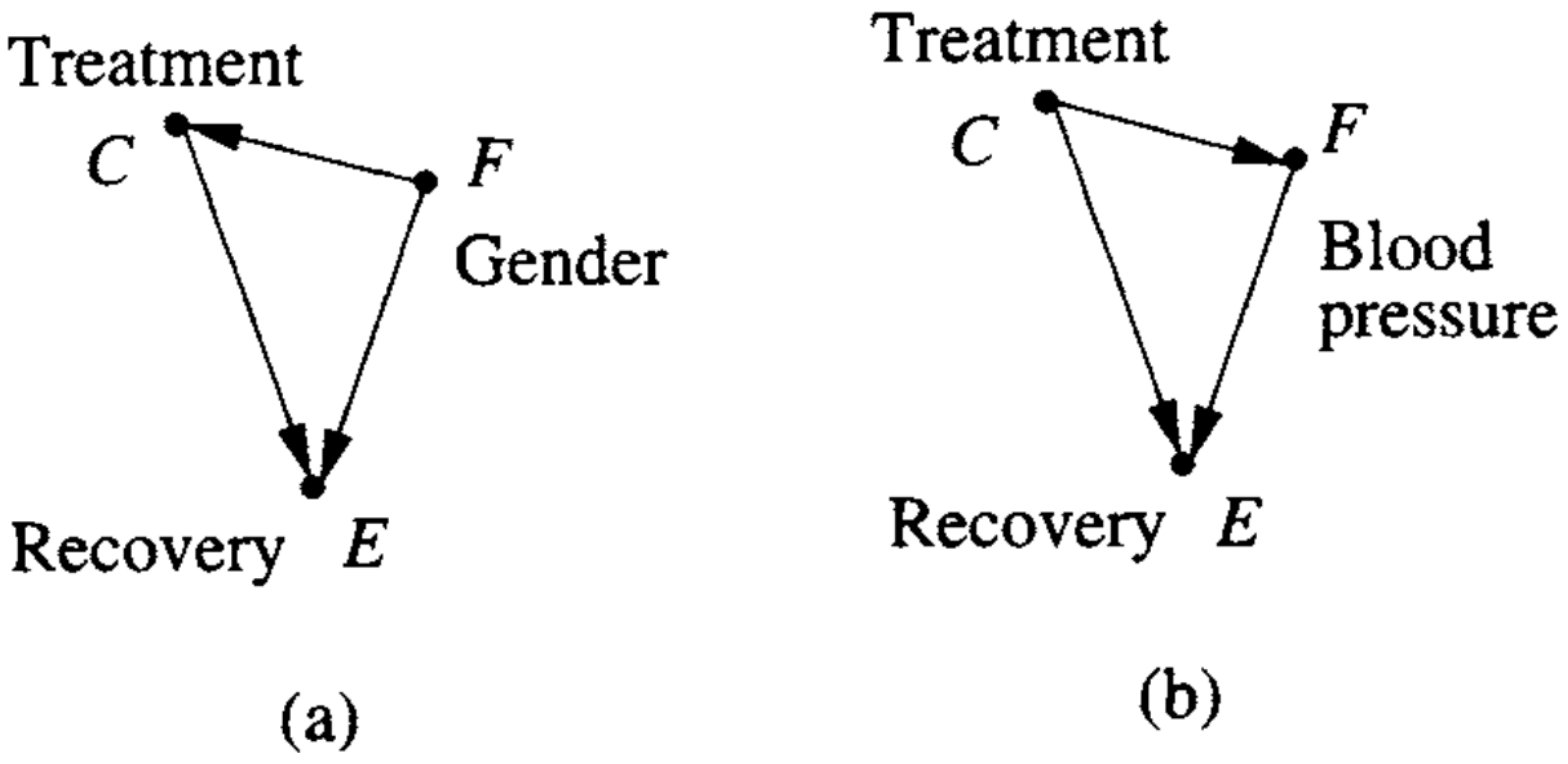
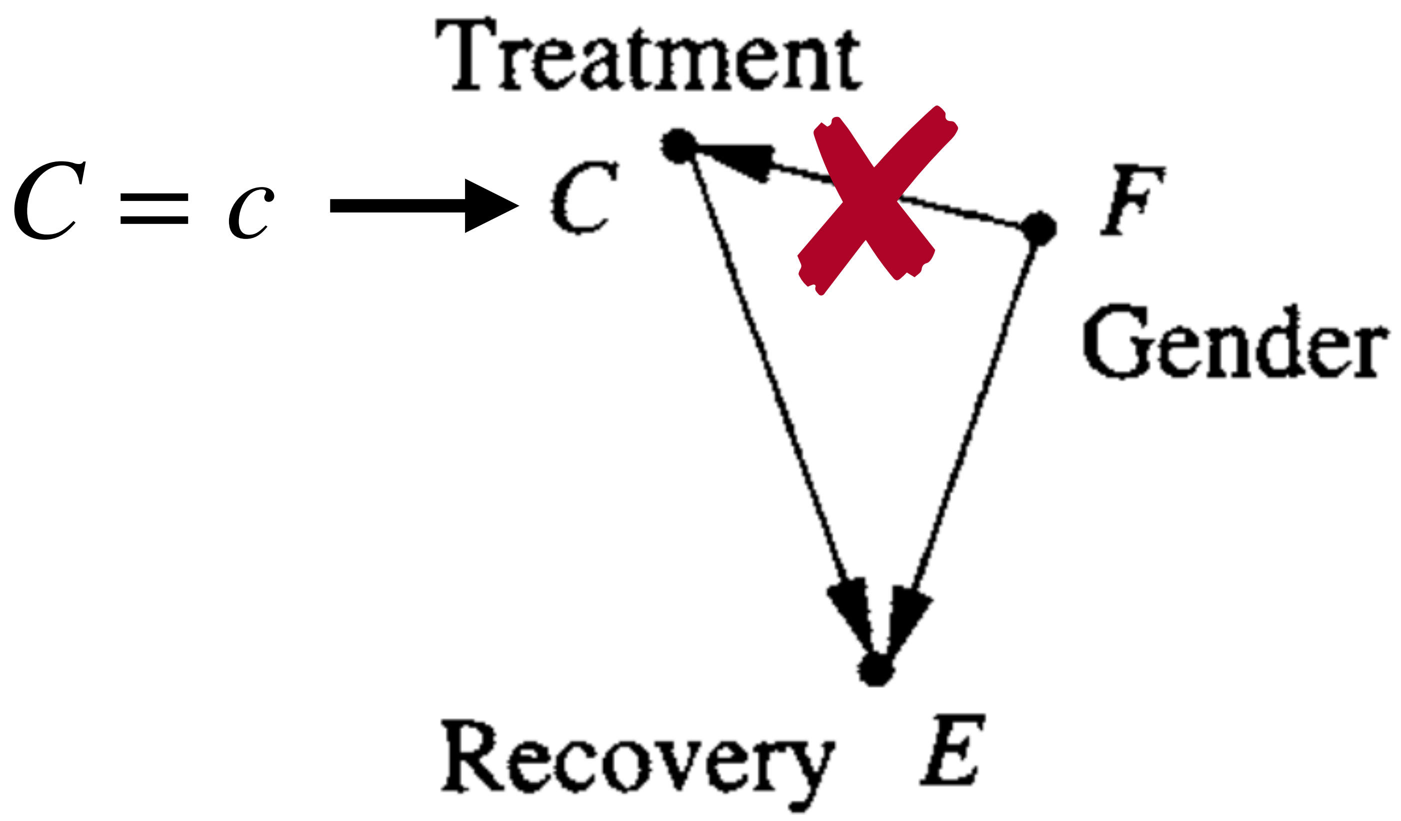


Conclusion: Our actions shall not be based on statistical consideration!

Role of interventions



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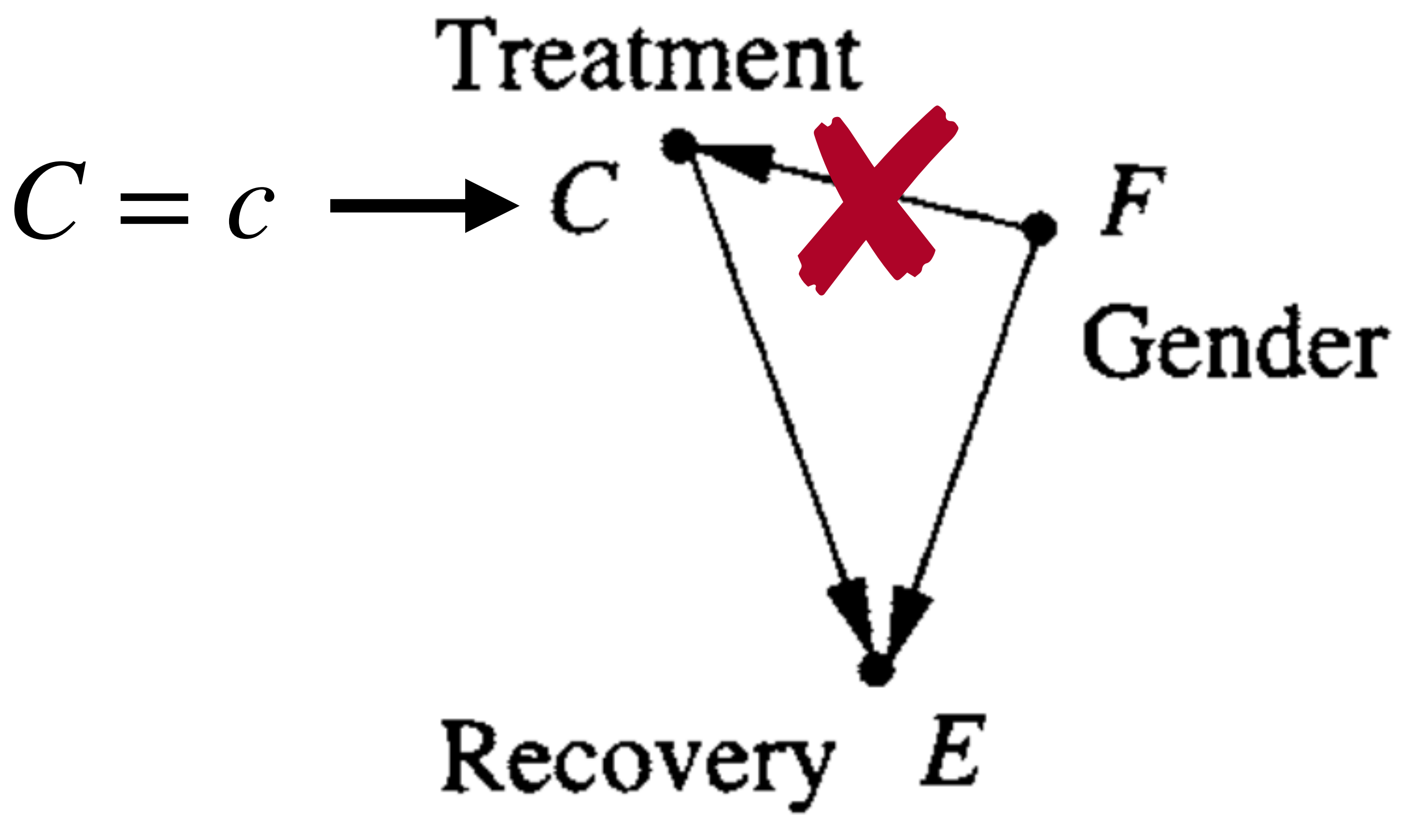

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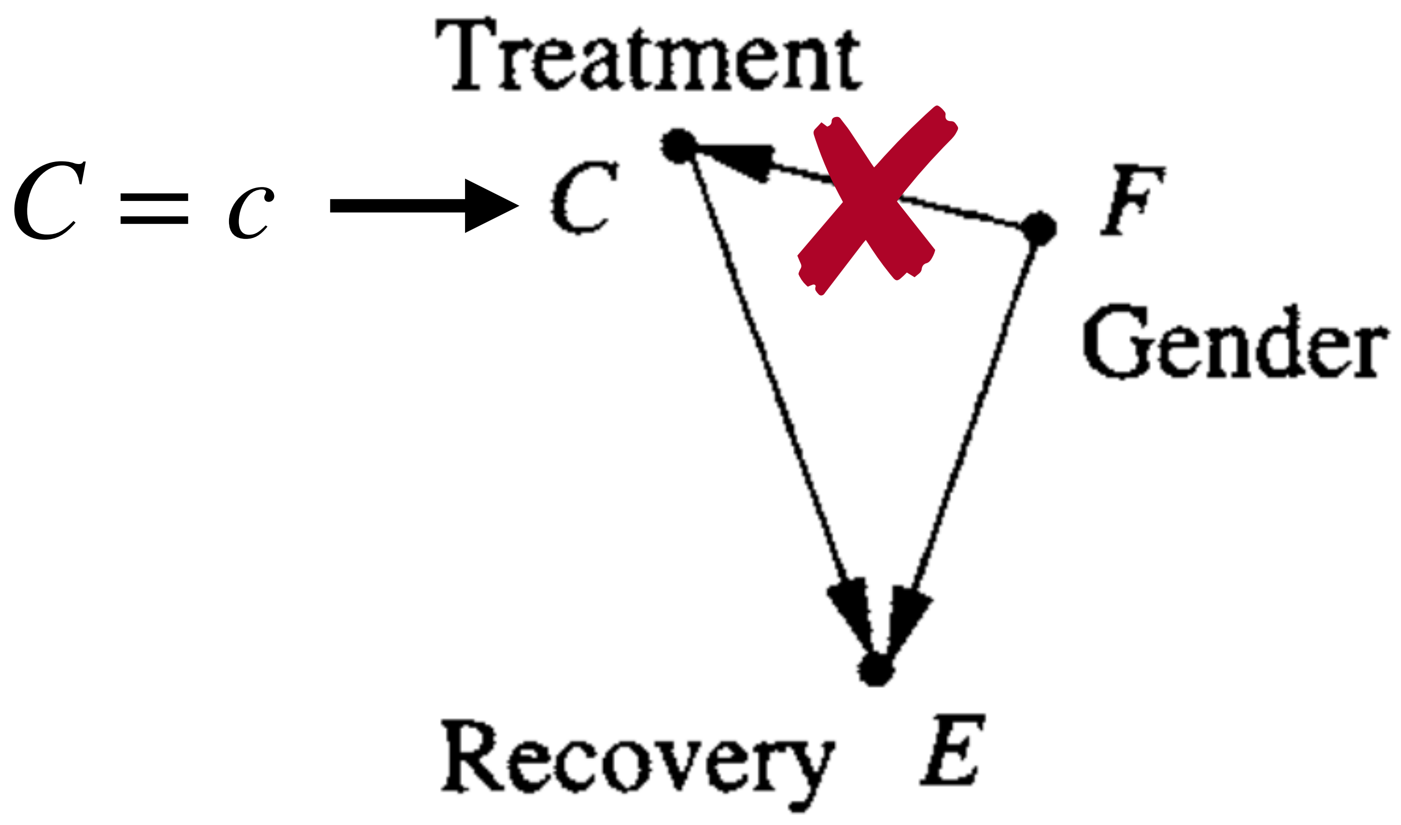

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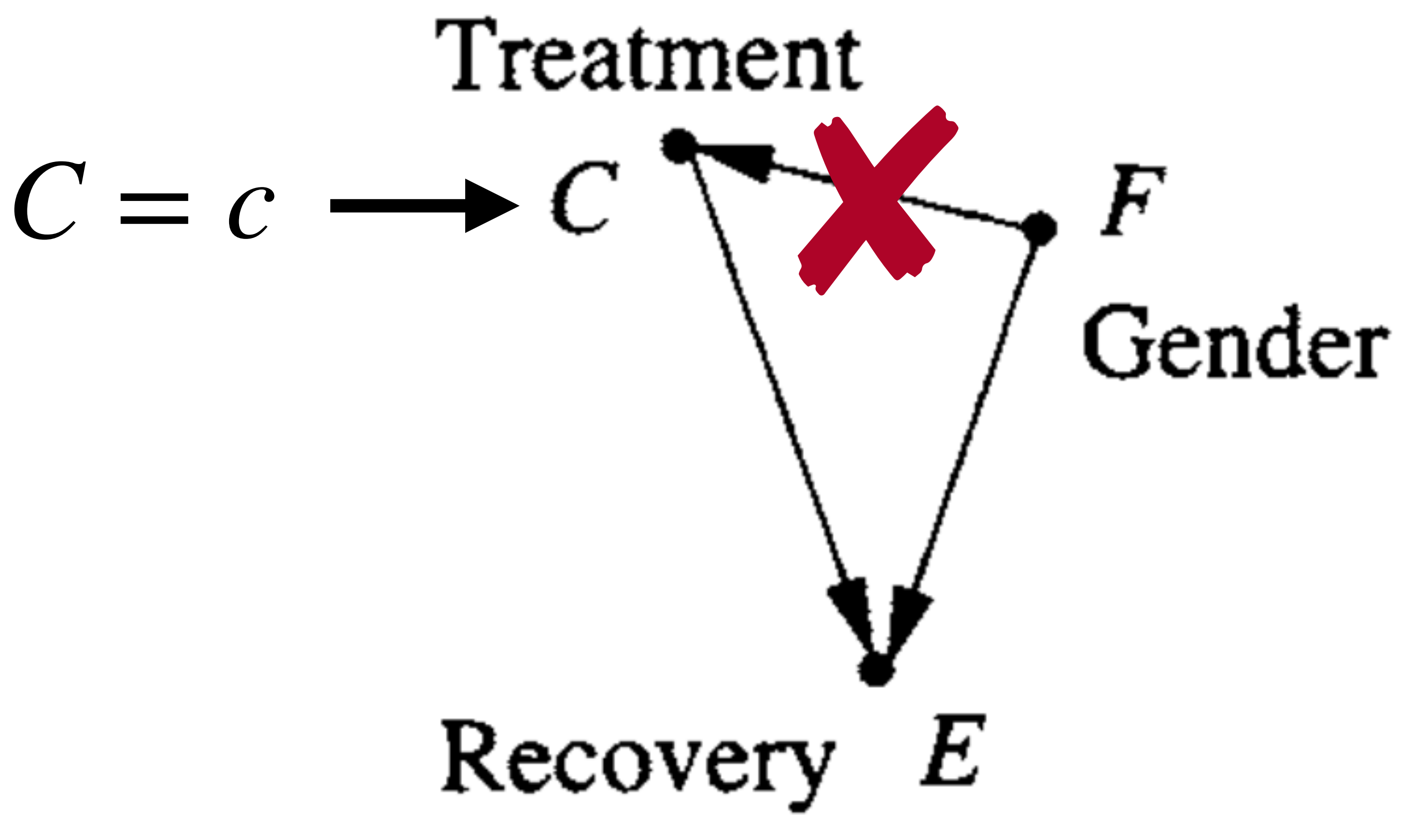

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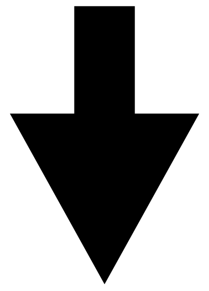

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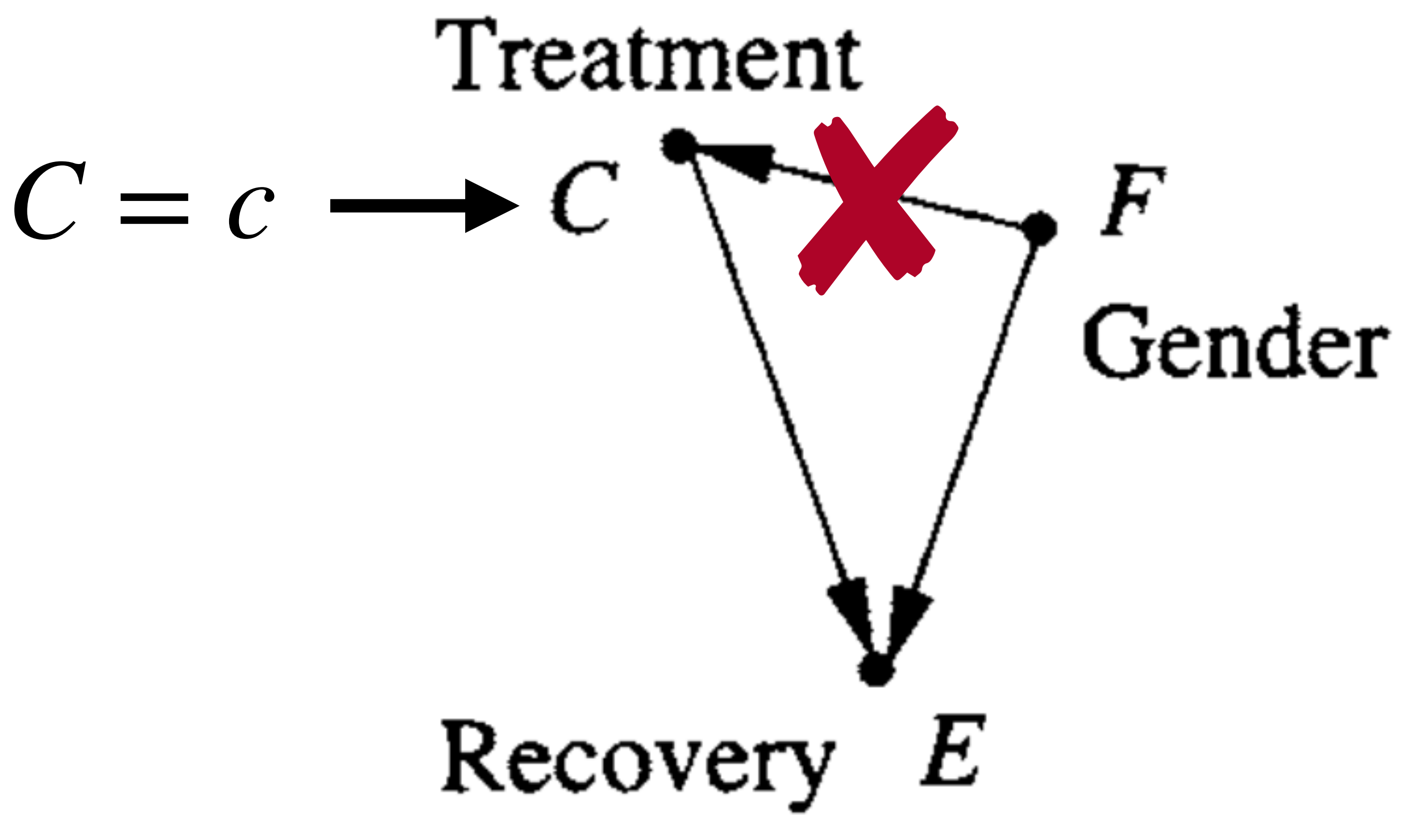

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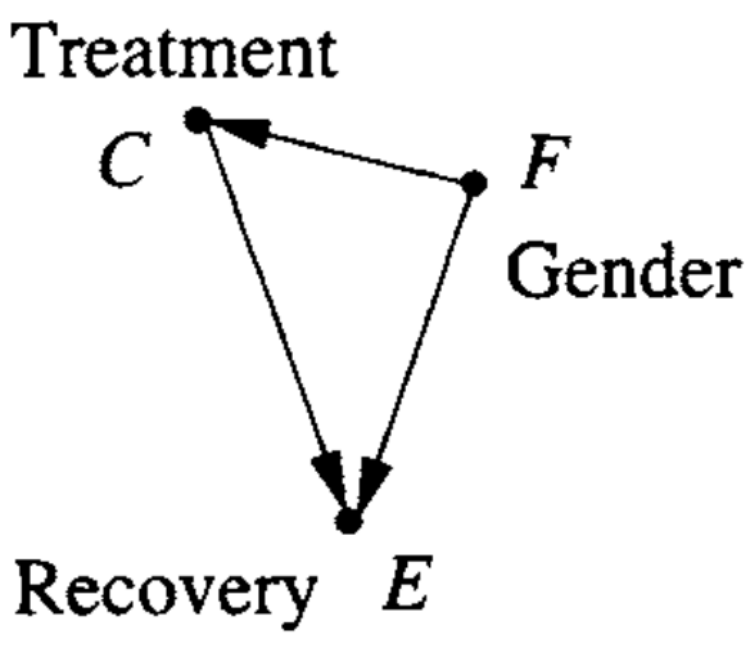

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Understanding Simpson's paradox

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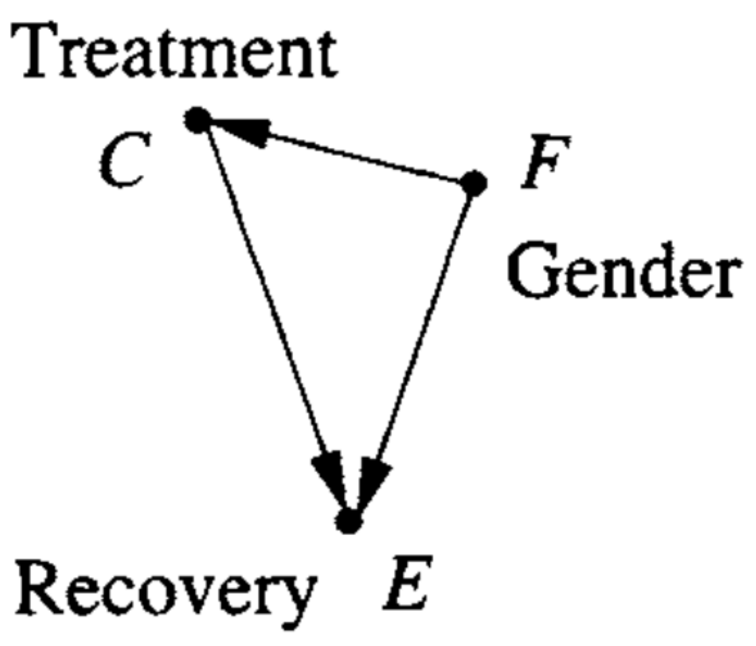

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A path is blocked by F if and only if it contains,

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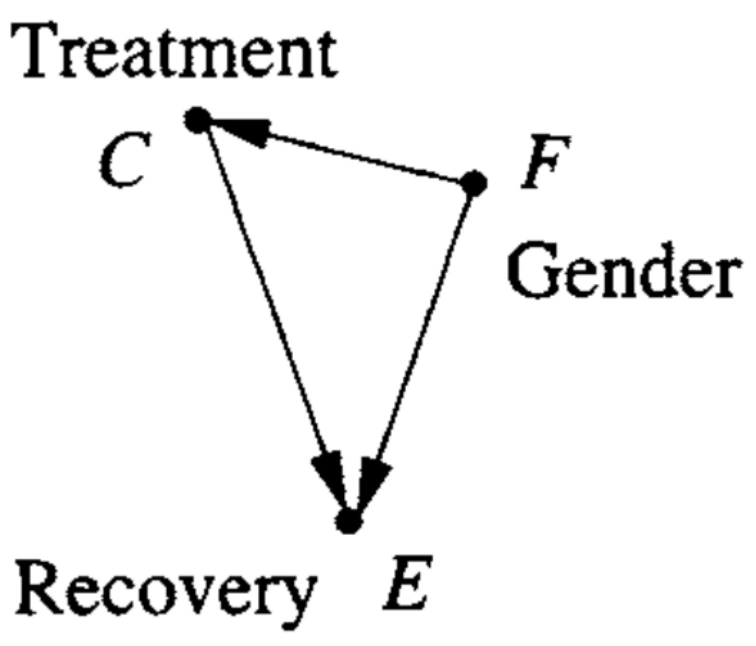

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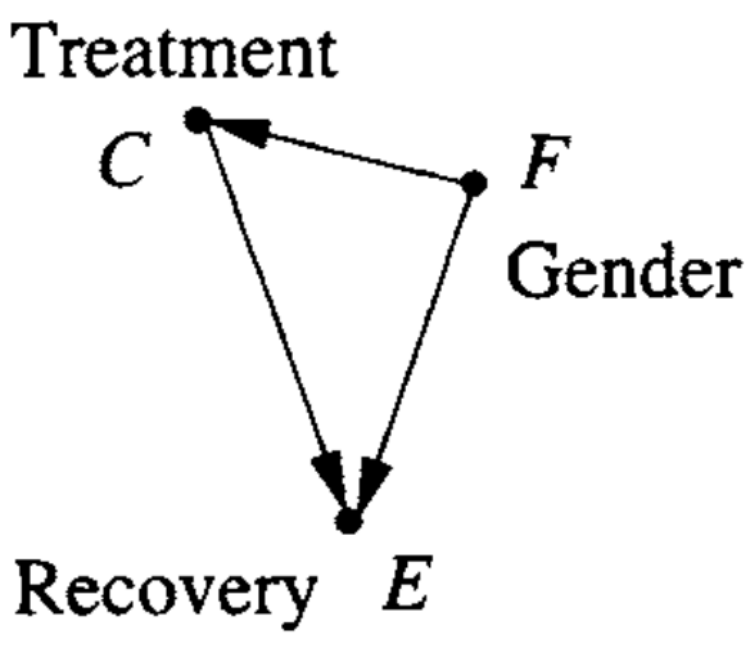

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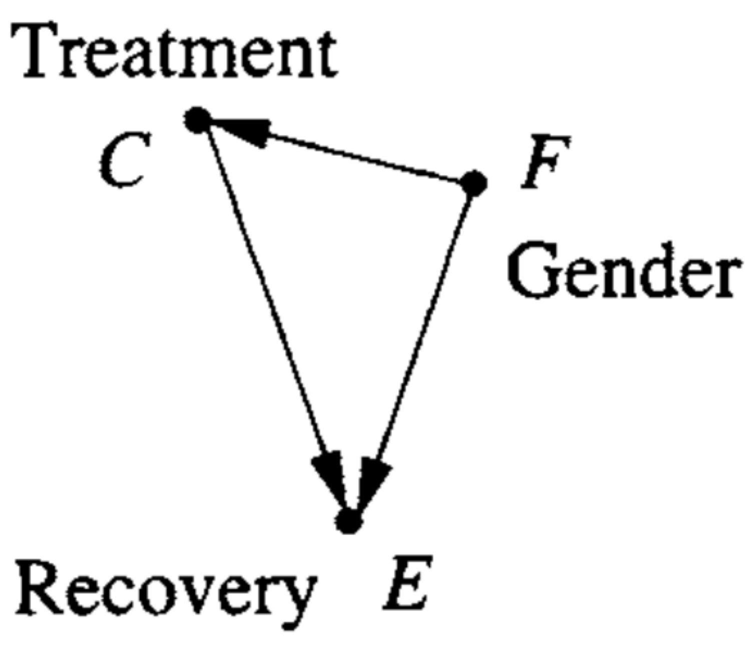

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1. F is not a descendant of C
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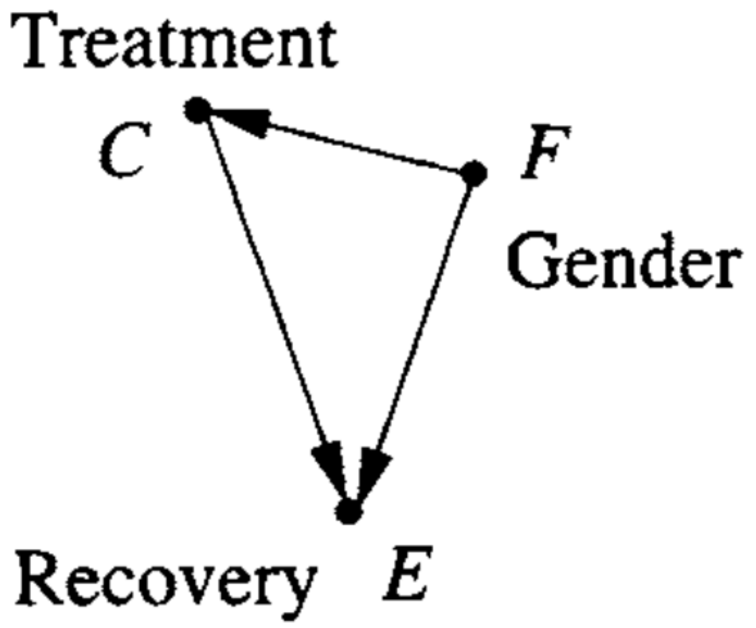
Understanding Simpson's paradox



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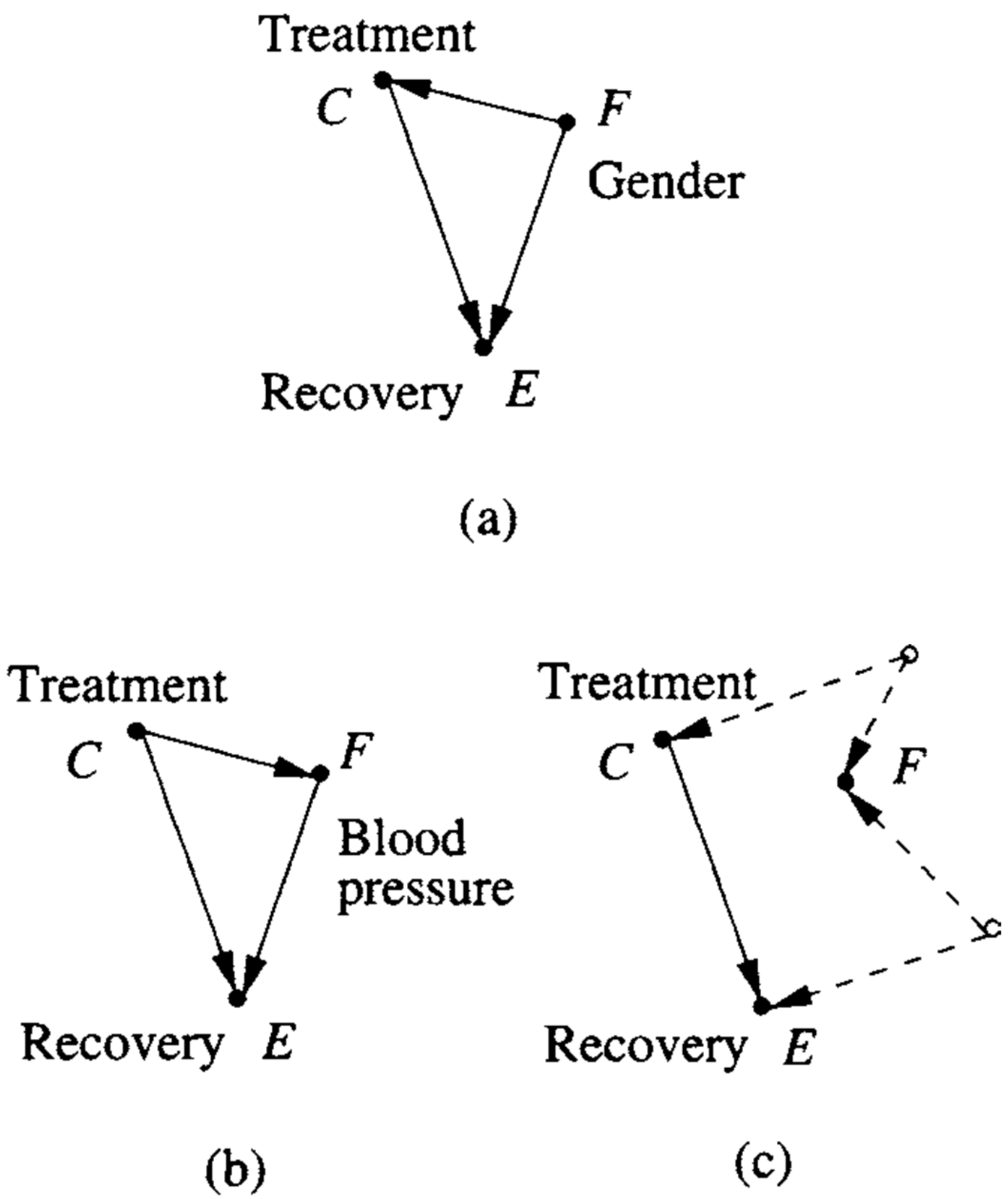

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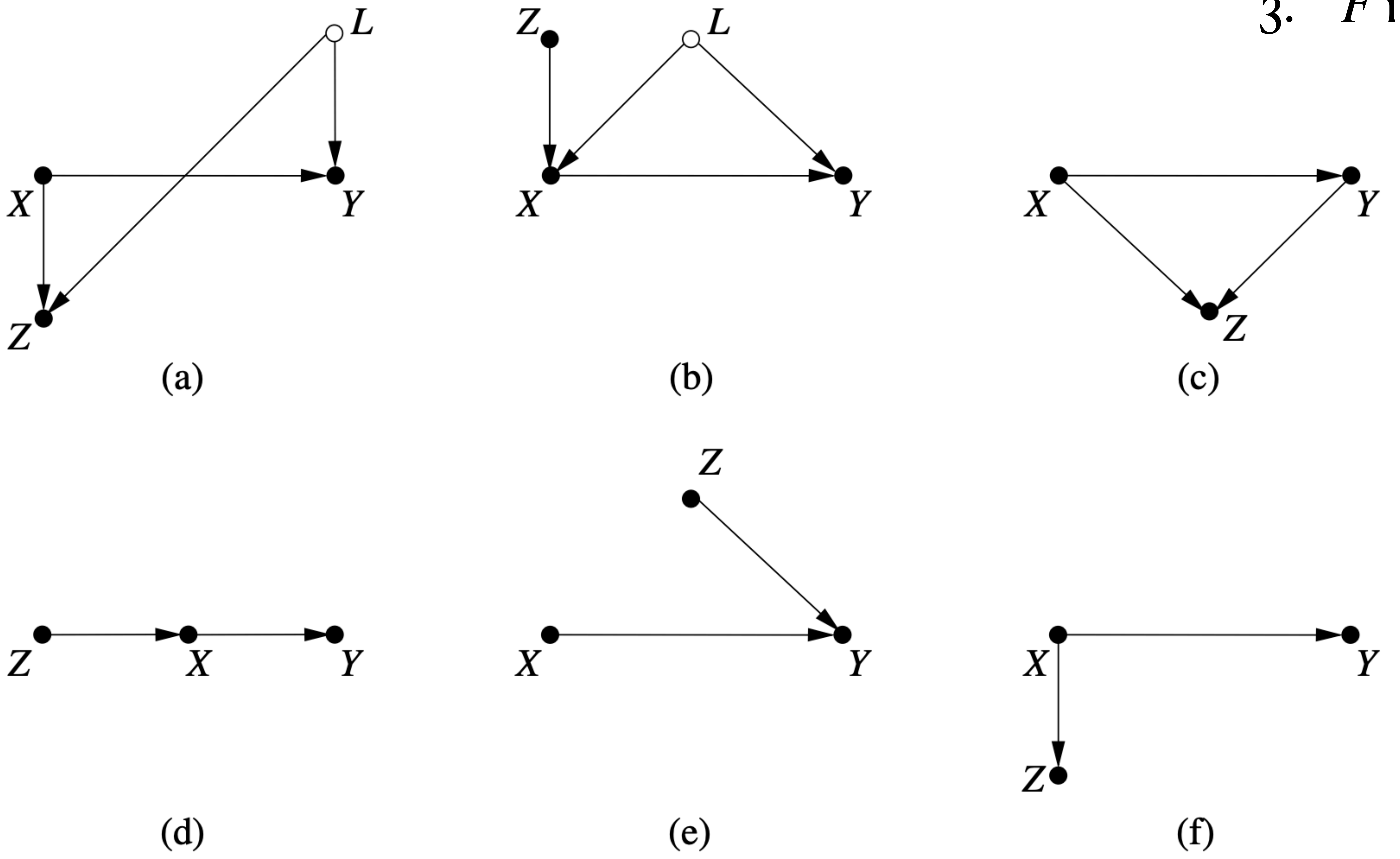

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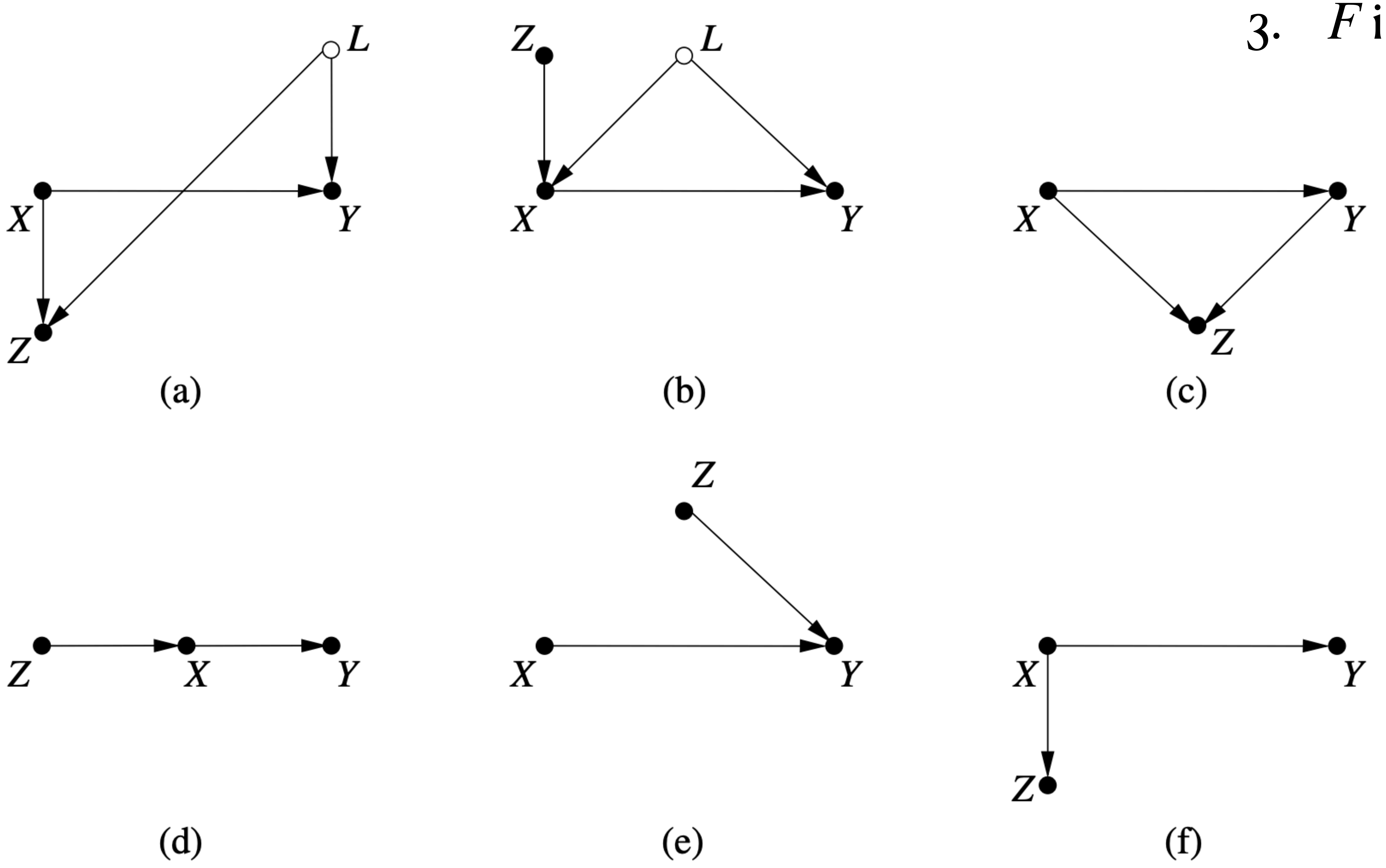
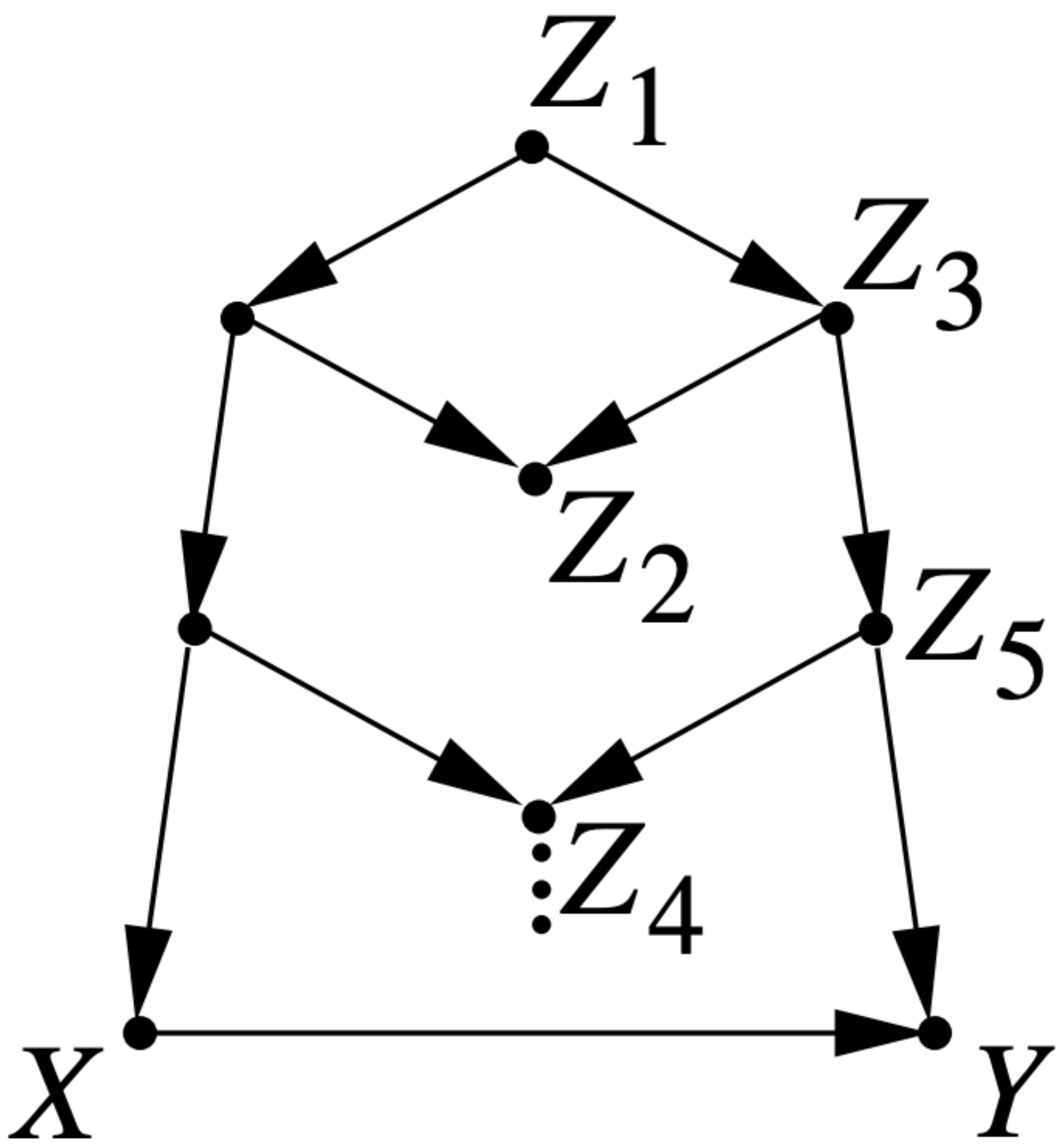


Bonus Question: In which of these causal structures is Simpson's reversal realisable?

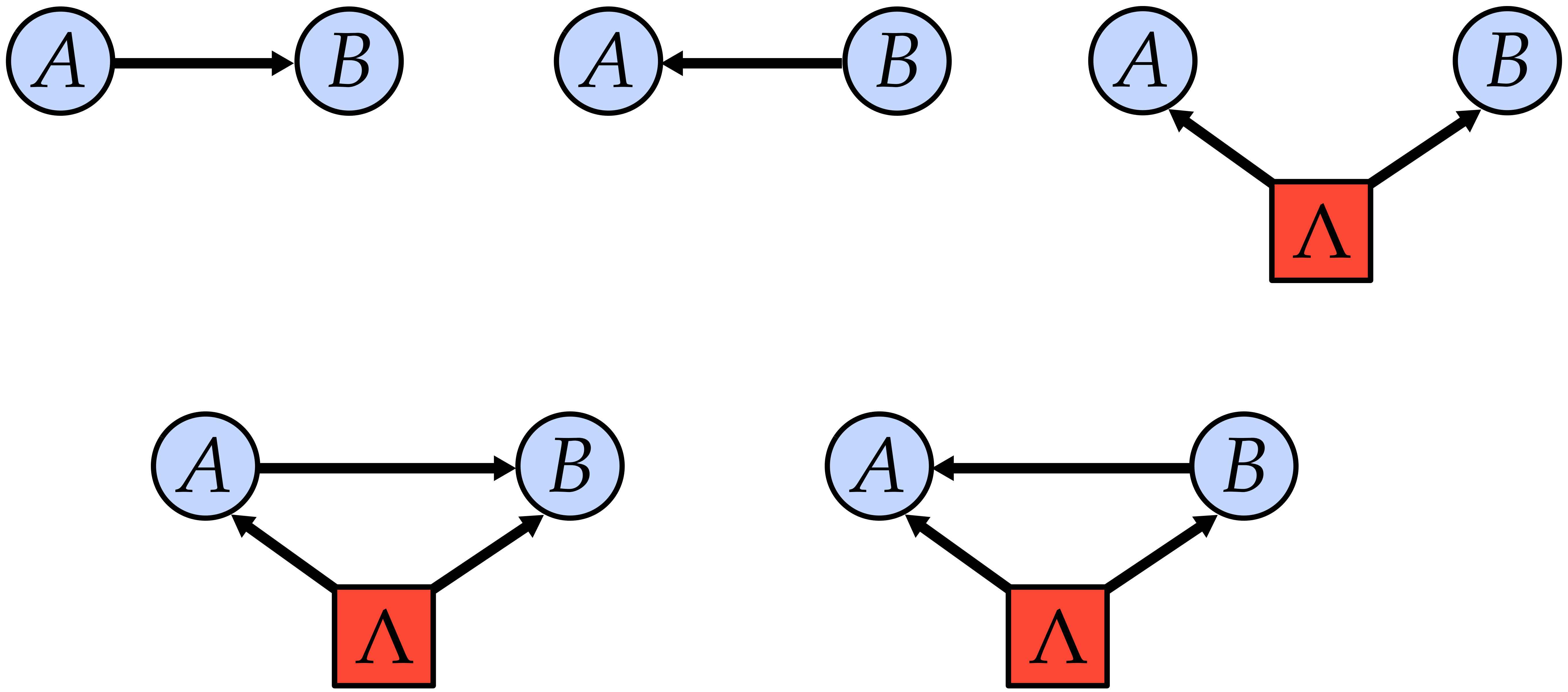
A multi-stage Simpson's paradox



Part 2:
Observational Causal Inference

Recap: Causal Reasoning

Causal Reasoning



Example: A – smoking, B – cancer, Λ – genetics

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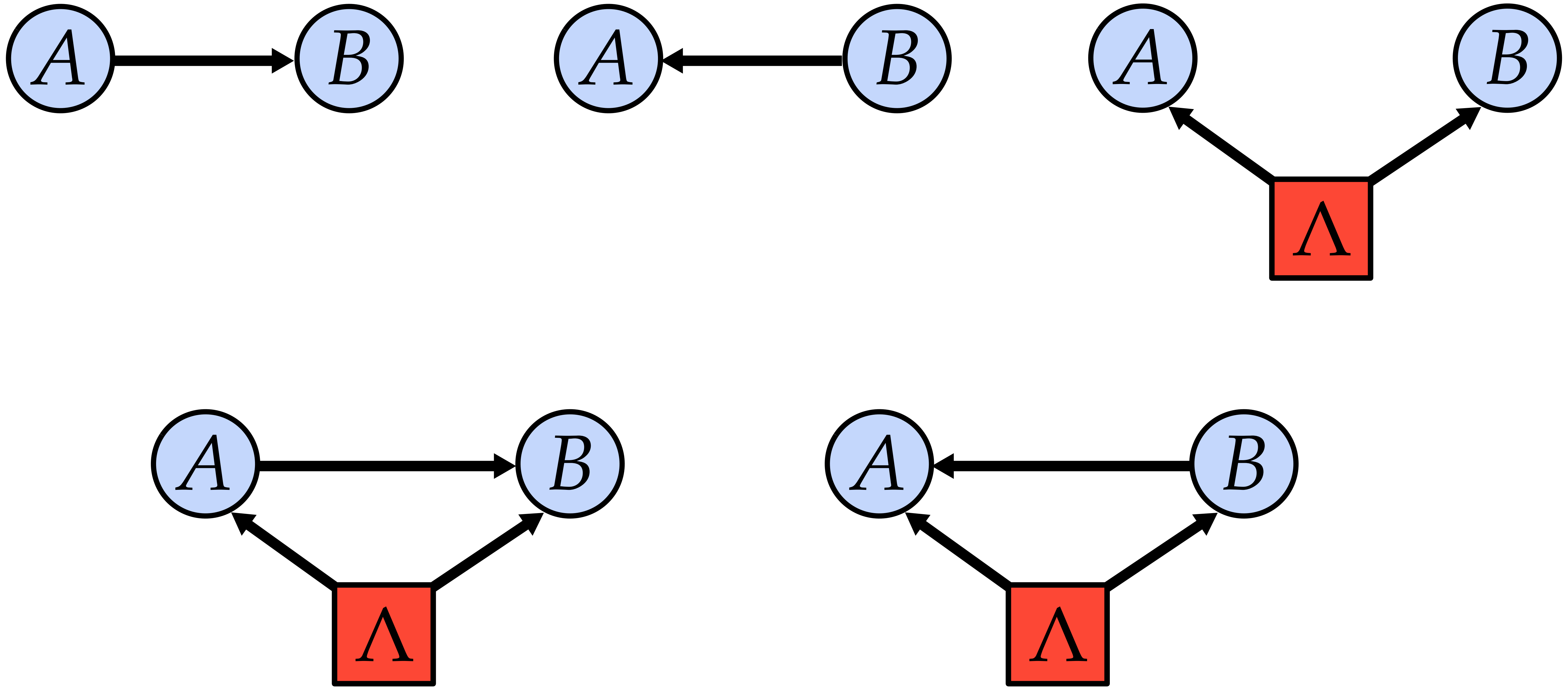

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Notations: A, B, Λ – random variables, a, b, λ – values, $p(a) = p(A = a)$.

Can we measure how much
A is influencing B?

Measuring Causality: Interventions

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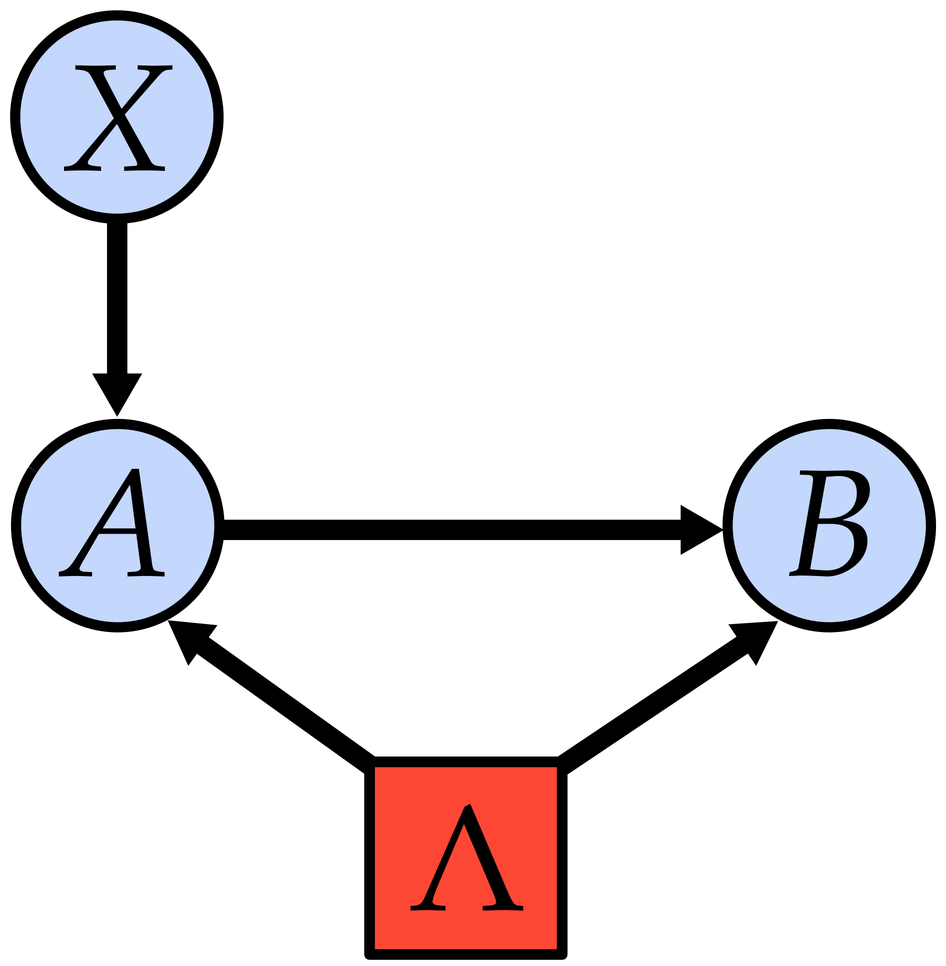
Average causal effect[1]:

$$\mathrm{ACE}_{A\to B} = \max_{a,a',b}\Big(p(b|do(a)) - p(b|do(a'))\Big). \tag{1}$$

Measuring Causality: Instrument



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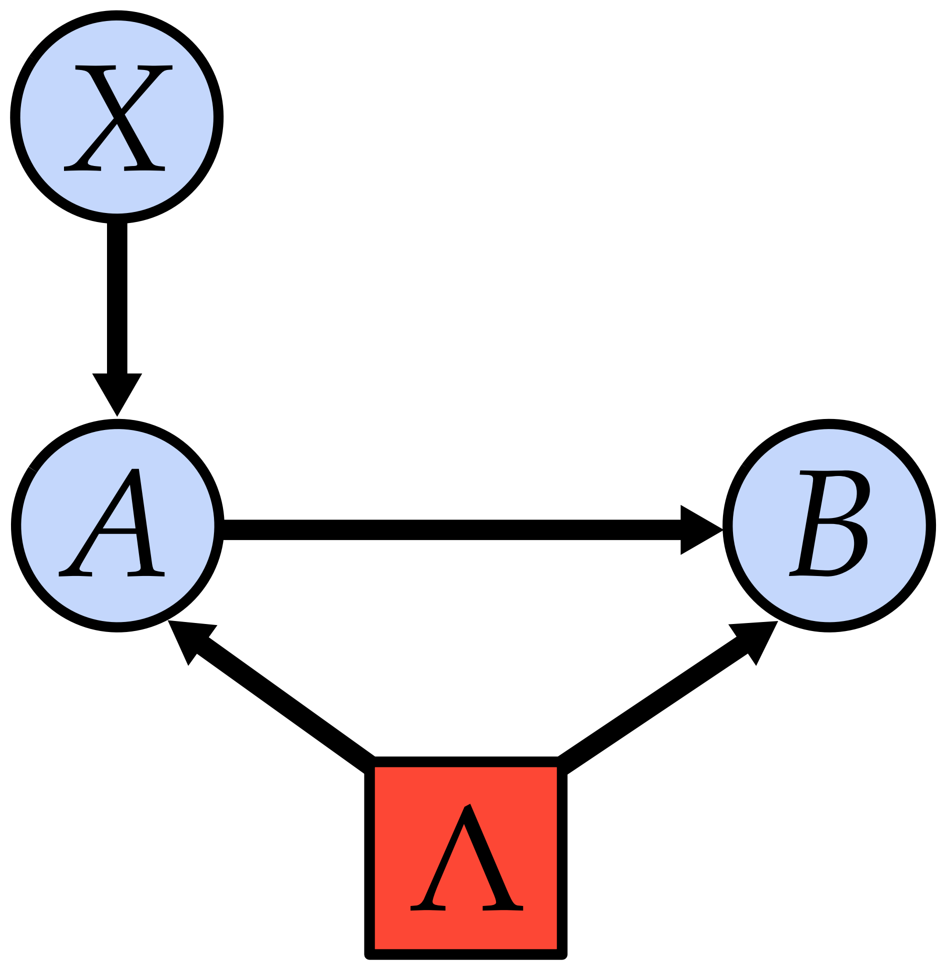

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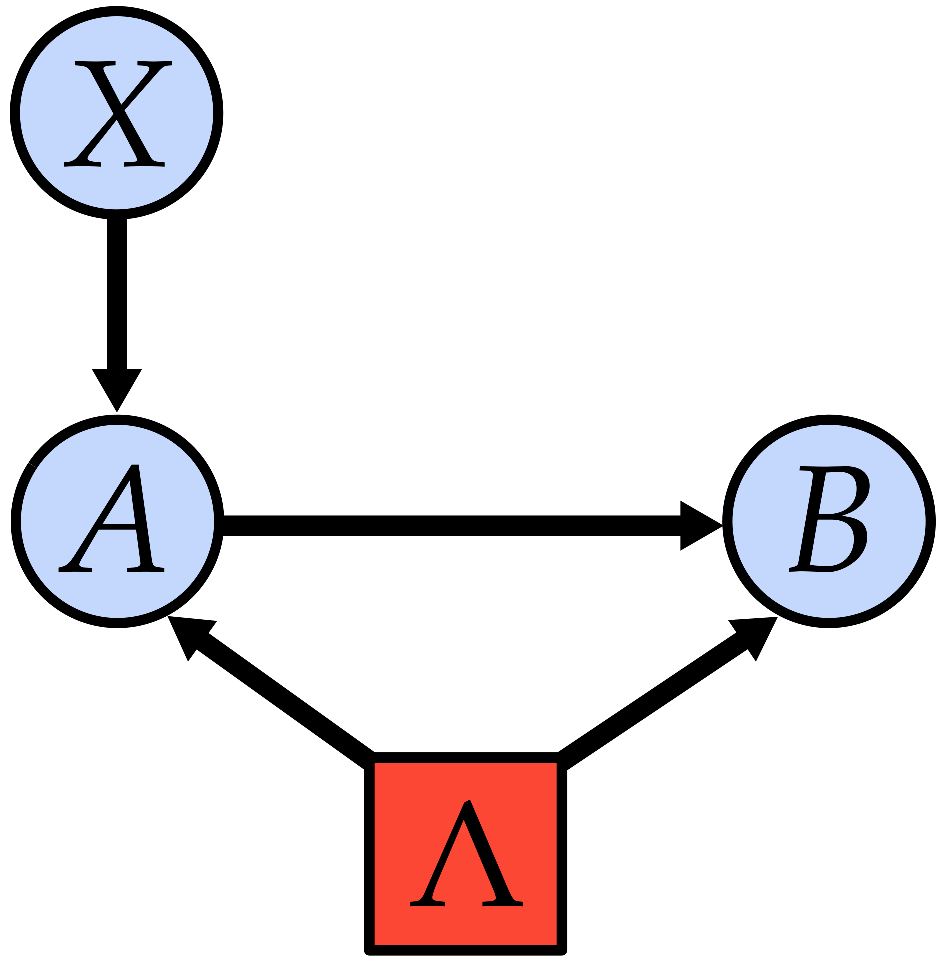

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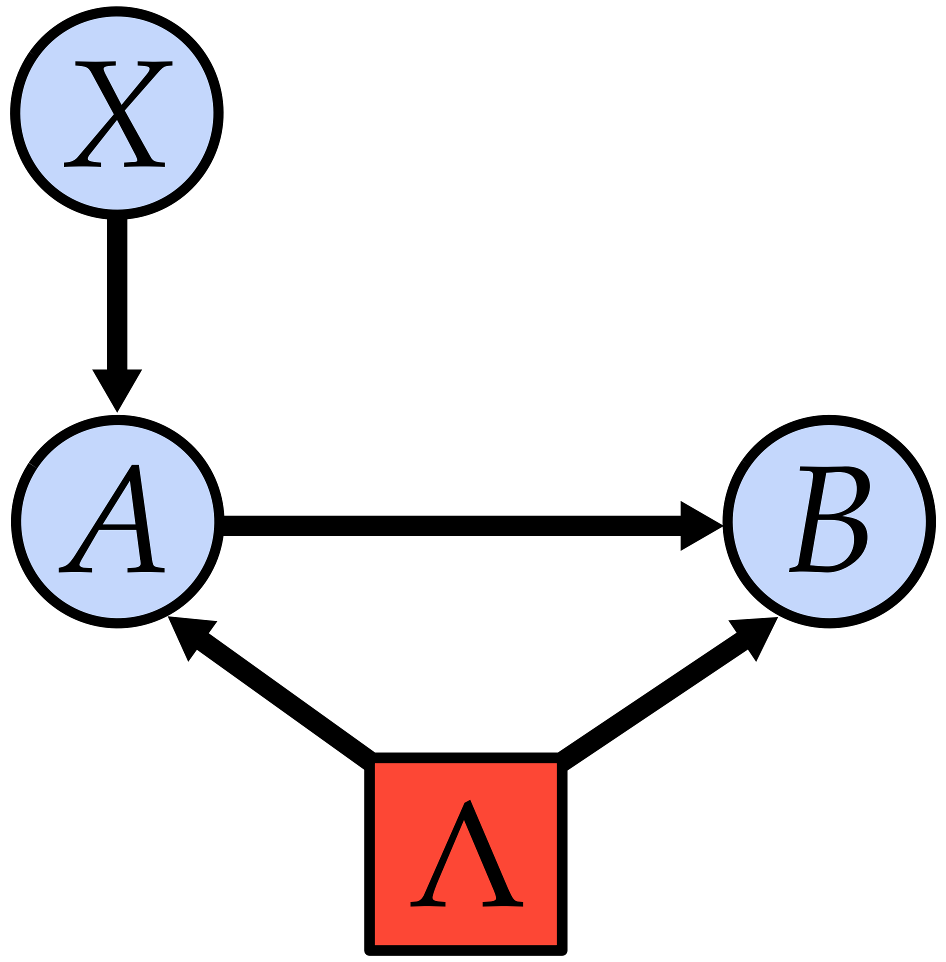

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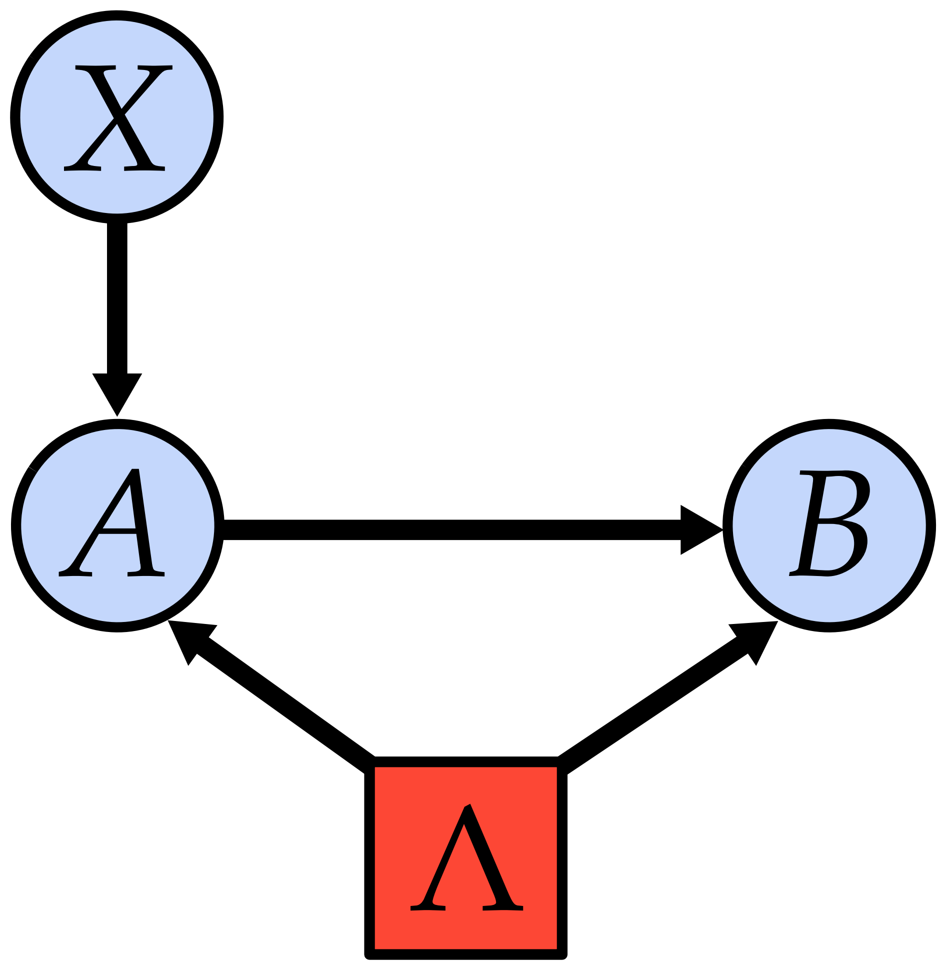
If $b = \kappa a + \lambda$, then $\kappa = \dfrac{C(X,B)}{C(X,A)}$, where $C(X,B) = \langle X, B\rangle - \langle X\rangle\langle B\rangle$ – covariance, and $C(X,\Lambda) = 0$.

Simplest Instrumental Scenario



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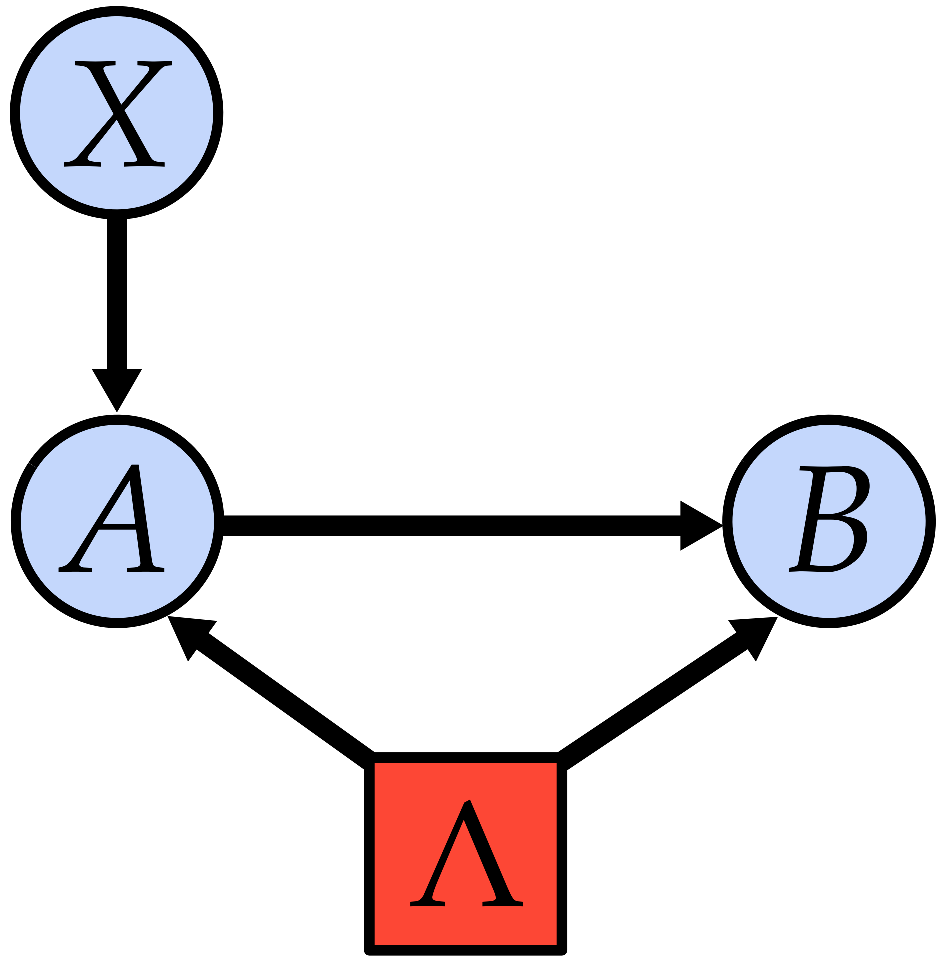

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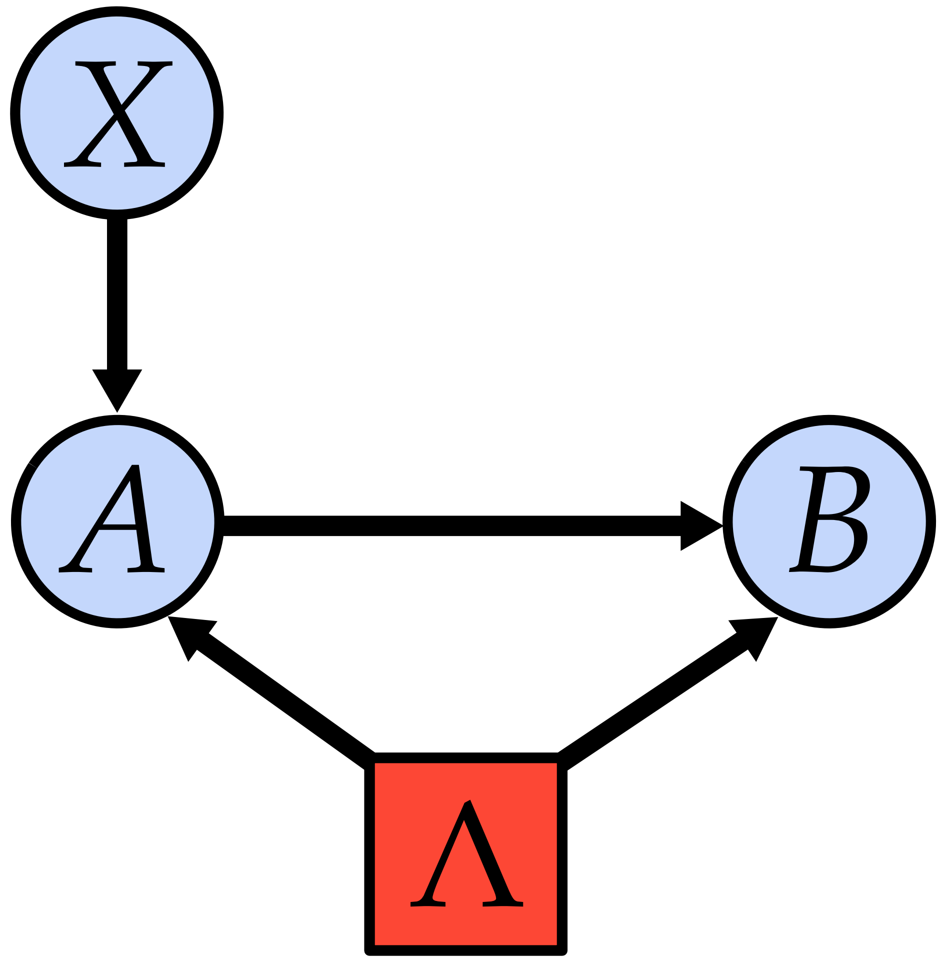

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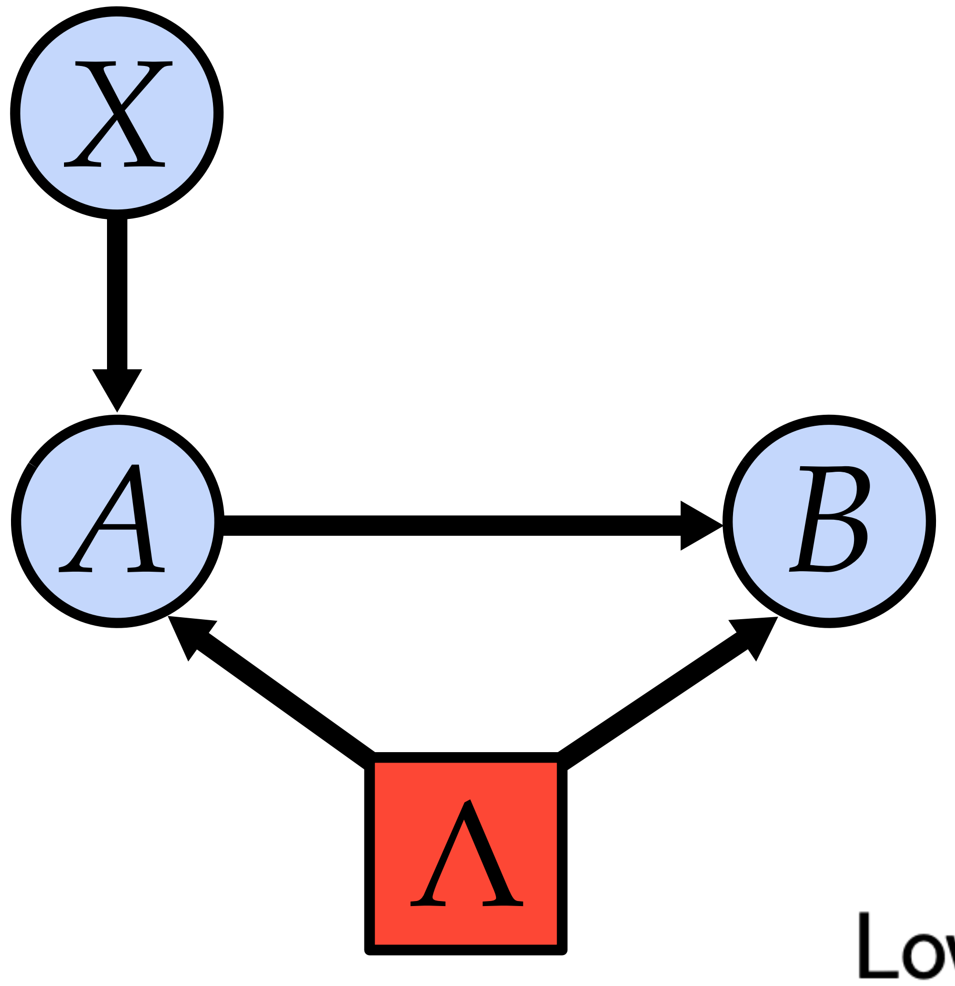
$x, a, b \in \{0, 1\}$

$p(a, b|x)$

Simplest Instrumental Scenario



Example: A – smoking, B – cancer, Λ – genetics, X – taxation of tobacco.

$x, a, b \in \{0, 1\}$

$p(a, b|x)$

Lower bound[2]:

$$\mathrm{ACE}_{A\to B} \geq 2p(0,0|0) + p(1,1|0) + p(0,1|1) + p(1,1|1) - 2, \quad (3)$$

where

$$\mathrm{ACE}_{A\to B} = \max_{a,a',b} \Big(p(b|do(a)) - p(b|do(a')) \Big).$$

[2] A. Balke and J. Pearl, "Bounds on treatment effects from studies with imperfect compliance," Journal of the American Statistical Association 92, 1171–1176 (1997).

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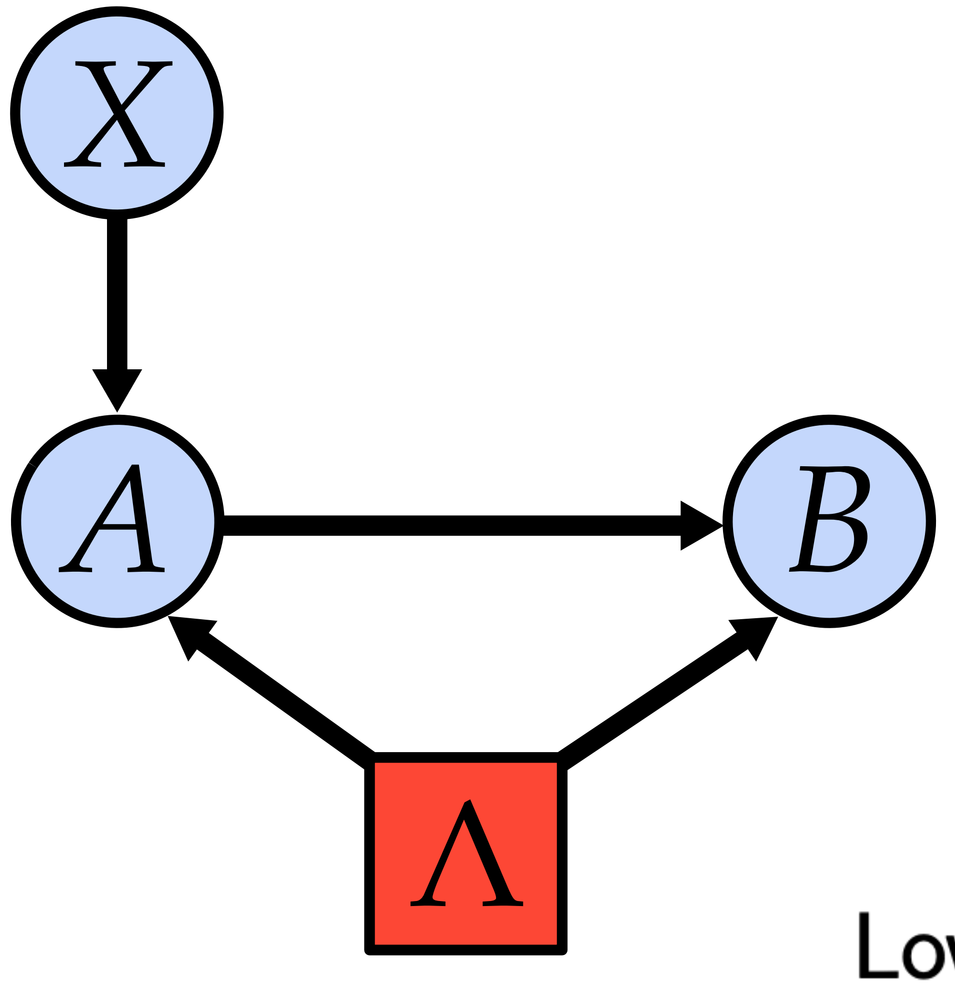

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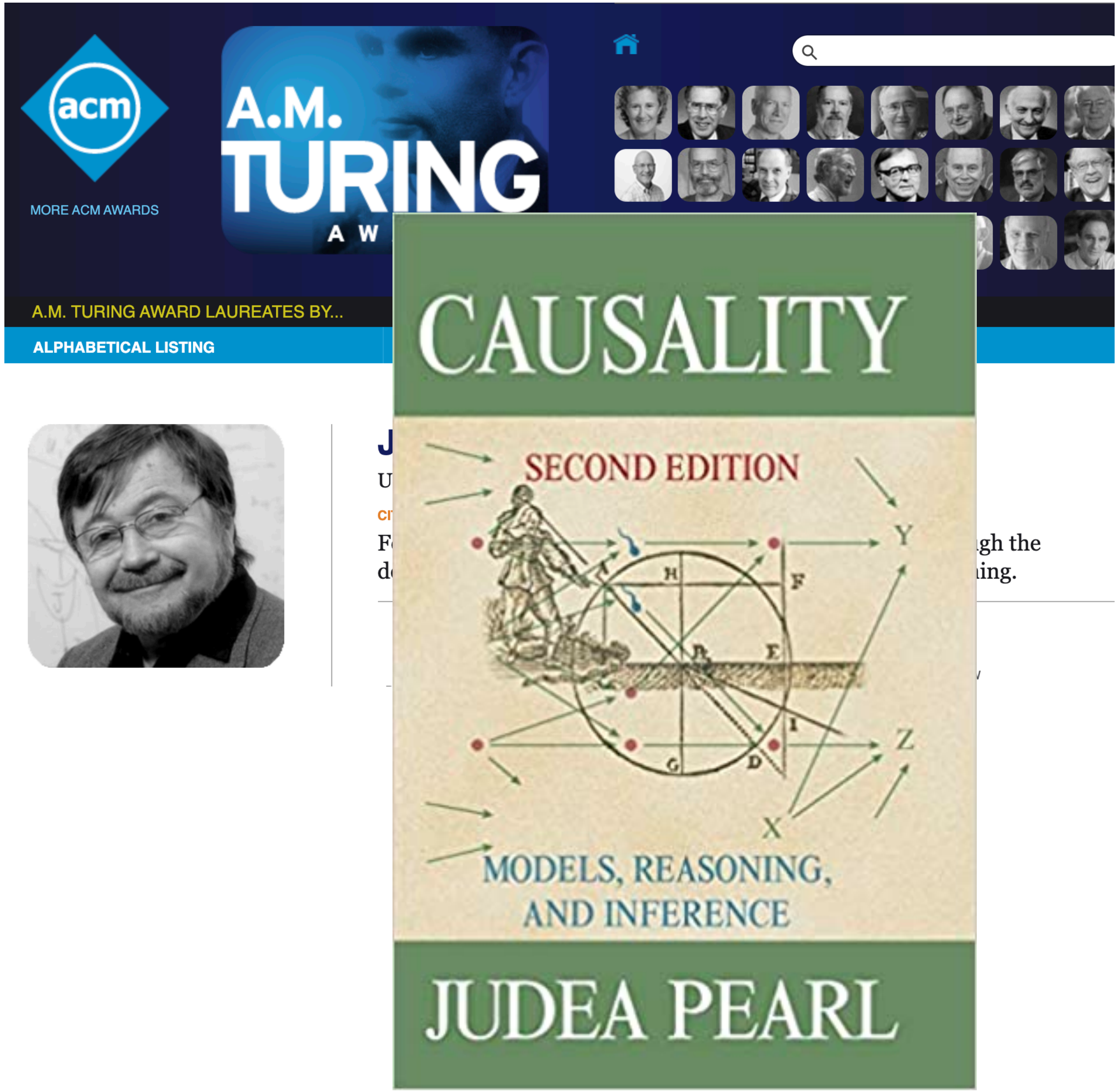
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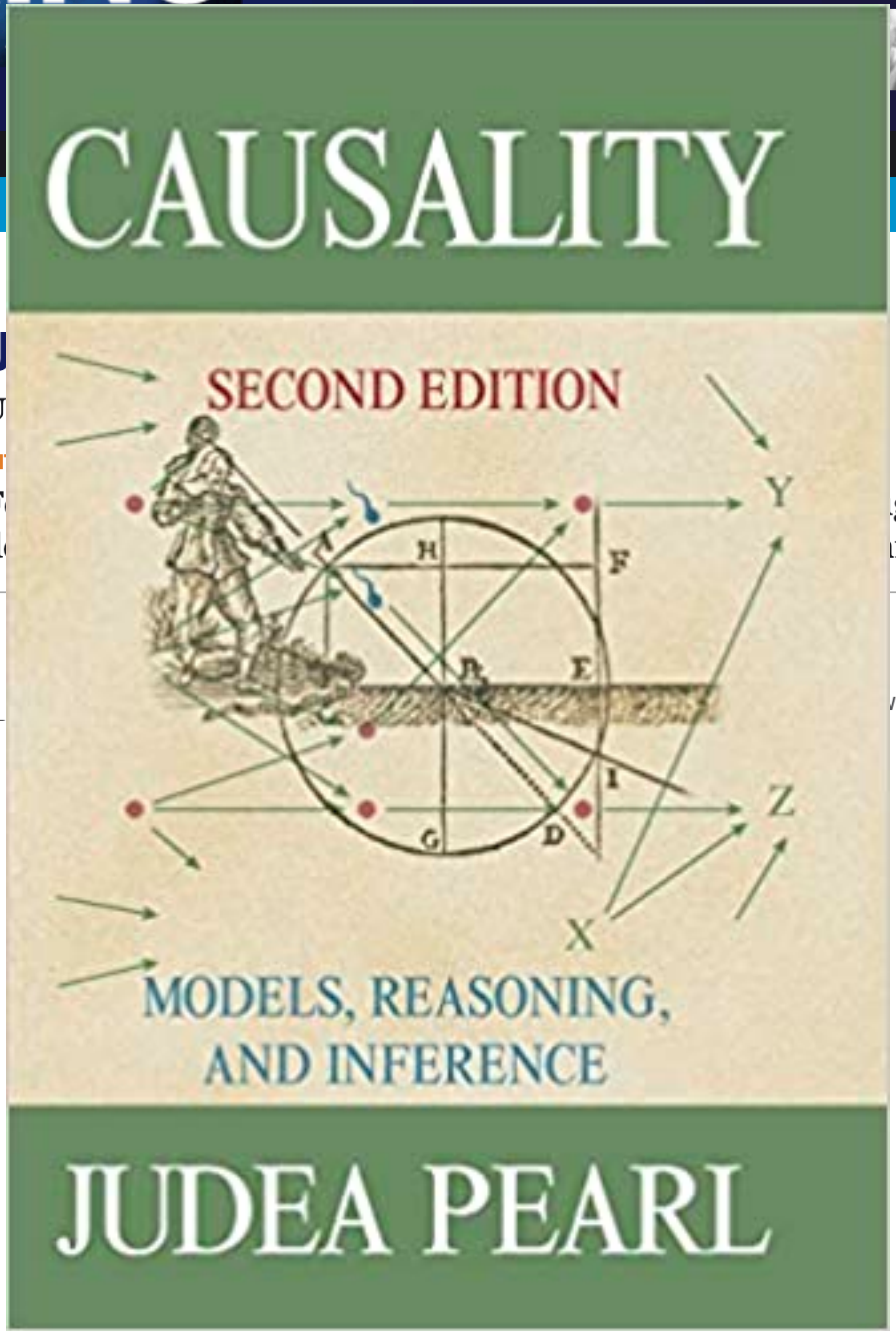
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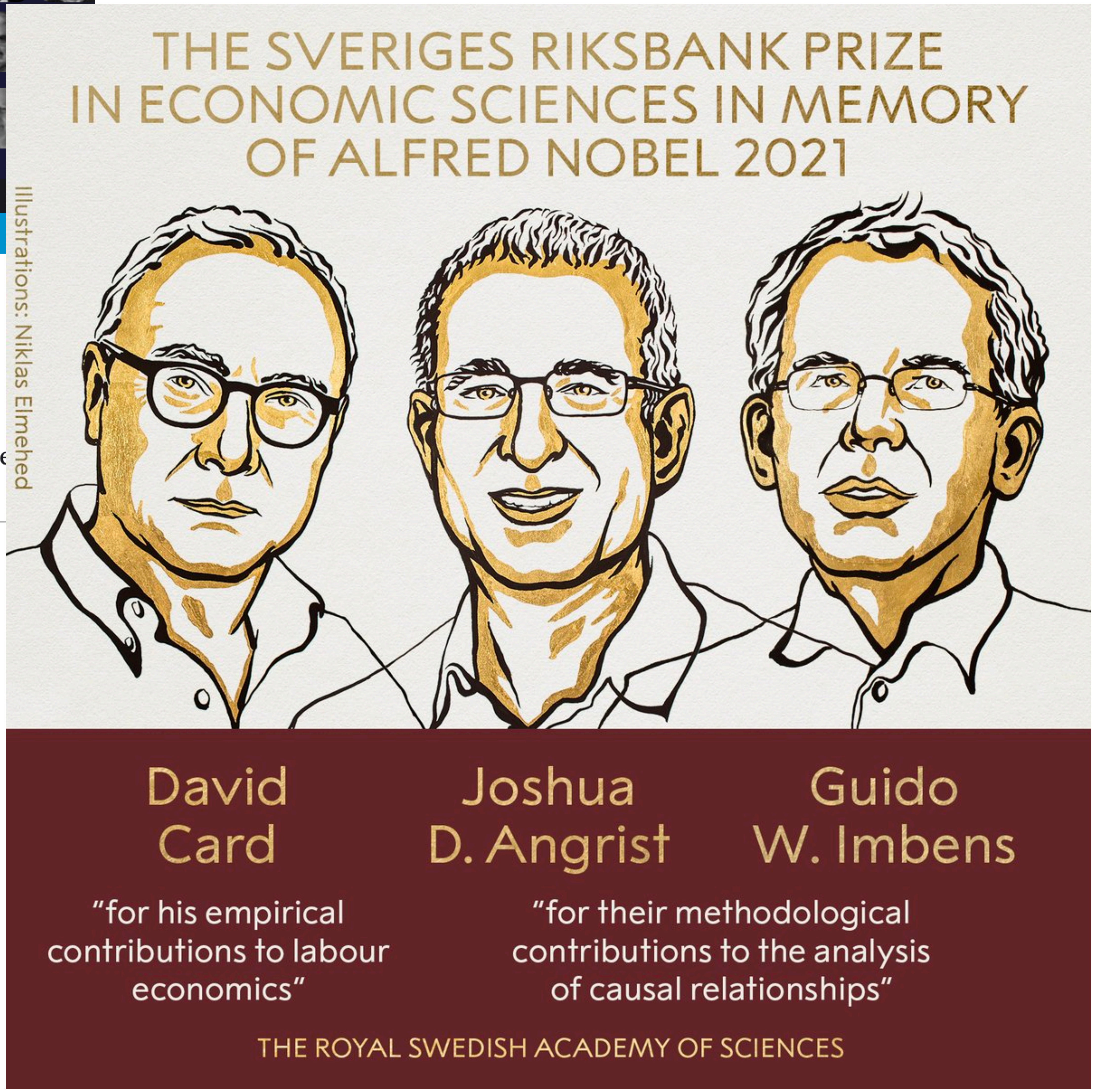
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OF ALFRED NOBEL 2021
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David
Card
Joshua
D. Angrist
Guido
W. Imbens
"for his empirical contributions to labour economics"
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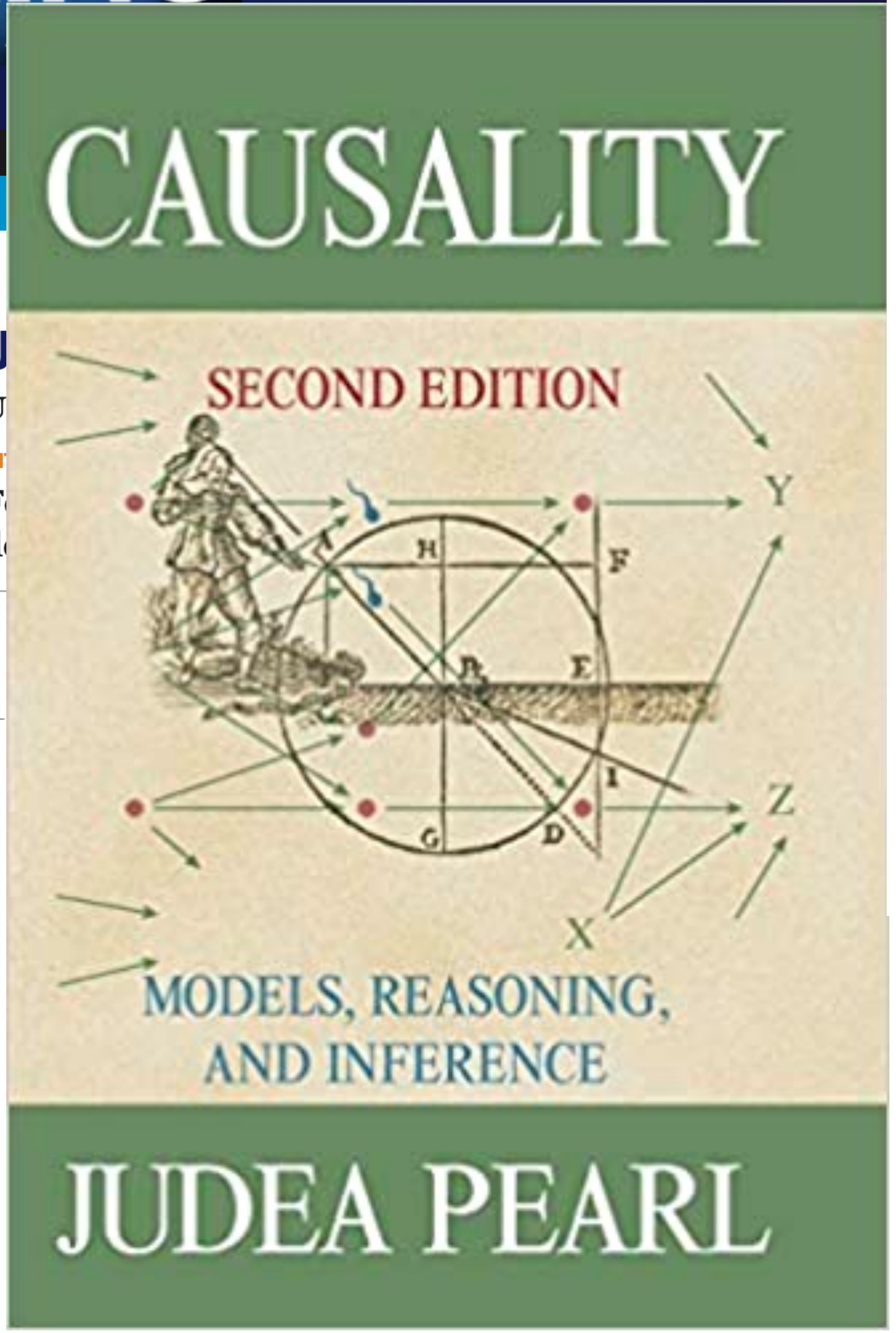

Josh Angrist

MIT
Verified email at mit.edu
Labor Economics Econometrics

FOLLOW

TITLE	CITED BY	YEAR
Mostly harmless econometrics JD Angrist, JS Pischke Mostly Harmless Econometrics	18941	2008
Identification of causal effects using instrumental variables JD Angrist, GW Imbens, DB Rubin Journal of the American statistical Association 91 (434), 444-455	6450	1996
Identification and estimation of local average treatment effects J Angrist, G Imbens National Bureau of Economic Research	5437	1995
Does compulsory school attendance affect schooling and earnings? JD Angrist, AB Keueger The Quarterly Journal of Economics 106 (4), 979-1014	3321	1991
Instrumental variables and the search for identification: From supply and demand to natural experiments JD Angrist, AB Krueger	2804	2001

David Card — "for his empirical contributions to labour economics"

Joshua D. Angrist, Guido W. Imbens — "for their methodological contributions to the analysis of causal relationships"

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Conclusions

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Part 1:

Simpson's paradox and its resolution using the back-door criteria

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Simpson's paradox and its resolution using the back-door criteria

Part 2:

Observational Causal Inference

Average causal effect and its estimates

Things that we skipped (maybe next week)

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- Bottlenecks of the instrumental scenario
- And how to treat them?

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